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Methods for Detecting Weak Light Signals

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Theoretical and experimental evaluation of three low-level photodetection techniques, namely, lock-in, noise-voltage, and electron-pulse counting are presented. The 992 cm^{-1} Raman band of benzene excited by a 4.2-mW He–Ne laser operating at 6328 Å provided a weak light signal which was detected by an EMI 9558 A, cooled photomultiplier. The electron-pulse-counting method employing a pulse-height discriminator was found to be superior to the other methods with regard to both signal sensitivity and signal-to-noise ratio.

Index Headings: Detection; Raman spectra; Laser.

THIS paper is concerned with the evaluation of methods for detecting light signals having powers less than 10^{-14} W. The photodetection methods presented here should be useful for the detection of such low powers in spectral regions accessible to photomultipliers. These methods should be of particular importance to the fields of Rayleigh, Brillouin, and Raman spectroscopy.

We have evaluated, both theoretically and experimentally, three detection schemes: (1) lock-in (phase sensitive), (2) noise-voltage, and (3) electron-pulse counting. Signal-to-noise expressions have been derived for these three techniques, as well as for dc detection. The signal-to-noise ratios of the 992 cm^{-1} vibrational Raman band in benzene have been measured by use of each of these methods. This band was excited with 4.2 mW of 6328 Å radiation from a He–Ne laser. The Raman scattering resulting from a single pass through a 4.5-cm benzene sample was focused on the entrance slit of a Hilger–Muller UVISIR double-prism monochromator having a spectral slit width of $\sim$60 cm^{-1}. The radiation was detected with a cooled EMI 9558 A photomultiplier operated at 1250 V. Experimental values of the signal-to-noise ratio were obtained at the Raman peak by opening and closing a shutter at the entrance slit of the spectrometer. The ratio of the average peak height to the average amplitude of the fluctuations about the peak was taken as the signal-to-noise ratio. In the lock-in and noise-voltage methods the quantity S/N refers to a voltage ratio.

LOCK-IN TECHNIQUE

The lock-in amplifier is essentially a narrow-band, phase-sensitive amplifier followed by a low-pass RC filter. It selects a band of frequencies of bandwidth B_f about a reference frequency, f_c, from a signal applied to its input, and converts the information to an equivalent bandwidth at zero frequency. The dc signal is amplified and passed on to a low-pass filter. The experimental setup is shown schematically in Fig. 1.

The signal-to-noise ratio of such an amplifier is[1]

$$S/N=(4V^2/W_0B_f)^{\frac{1}{2}}, \qquad (1)$$

where S and N are the rms values of the output signal and noise voltages, respectively, V is the rms value of the input signal voltage, and W_0 is the spectral power of the input noise at zero frequency. If the input circuit has a bandwidth B_s, the rms noise input voltage for a square noise band is[1]

$$V_N=(W_0B_s)^{\frac{1}{2}} \qquad (2)$$

which with Eq. (1) yields

$$S/N=(4V^2B_s/V_N{}^2B_f)^{\frac{1}{2}}. \qquad (3)$$

The bandwidth B_s is determined by the time constant of the photomultiplier circuit, i.e., $B_s=(R_LC)^{-1}$ where R_L is the load resistance, and C is the sum of the internal capacitance of the photomultiplier and the capacitance

[1] A. Van Der Ziel, *Noise* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1954), Ch. 13.

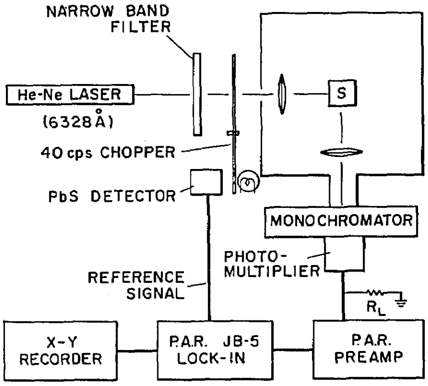

FIG. 1. Diagram of the lock-in detection technique for 90° Raman scattering.

of the cables to the amplifier. For an array of voltage pulses due to photoelectrons emitted from the cathode of the photomultiplier, the current through R_L consists of a dc dark current I_d and a dc signal current I_A. The input-signal voltage is then

$$V=R_LI_AM, \tag{4}$$

where M is an ac form factor determined by the shape of the modulation. $M\approx 1$ for the square-wave modulation used in our experiments. The noise voltage V_N due to shot noise is[2]

$$V_N\simeq R_L[2e(\mu I_A+\mu' I_d)B_s]^{\frac{1}{2}} \tag{5}$$

where e is the electronic charge, μ is the gain of the photomultiplier for signal current, and μ' is the gain of the photomultiplier for the dark current. We have taken $\mu\simeq\mu'$ since values of μ' are unavailable. It would be expected that $\mu'<\mu$ since some of the electrons contributing to dark current originate from the dynodes and consequently experience less multiplication. It must be noted that we have neglected the flicker $(1/f)$ noise contribution to the noise voltage. This component is expected to contribute to the noise spectrum below 1000 cps. Substitution of Eqs. (4) and (5) into (3) yields the signal-to-noise ratio in terms of the signal and dark currents as

$$S/N\simeq[2I_A{}^2/e\mu B_f(I_A+I_d)]^{\frac{1}{2}}. \tag{6}$$

The dependence of S/N on B_f given by Eq. (6) was experimentally verified. The dependence on I_A was found to be approximately linear. This implies that, for the intensities of Raman-scattered light obtained, I_A was significantly smaller than I_d. A typical experimental result is shown in Fig. 2(a).

To check the consistency of the signal-to-noise expressions of the various methods and to evaluate their sensitivity, it is necessary to obtain the values of the signal current and dark current. These values may be deduced from the experimental data of Fig. 2(a) using Eq. (6) together with the relation between output signal voltage and photomultiplier signal current

$$I_A=S/GR_L, \tag{7}$$

where G is the amplifier gain. Equation (6) may be solved for I_d to give

$$I_d=[2I_A{}^2/e\mu B_f(S/N)^2]-I_A. \tag{8}$$

From the data of Fig. 2(a) the output rms signal $S=3.1$ V, the peak-to-peak output noise voltage $=0.38$ V which, under the assumption of a gaussian distribution, corresponds to an rms noise voltage $N=0.075$ V and hence a signal-to-noise ratio $S/N=41$. These values, with the parameters $G=5\times10^4$, $R_L=10^6$ Ω, $\mu=10^6$, and $B_f=0.1$ cps, when substituted in Eqs. (7) and (8) give

$$I_A=6.0\times10^{-11}\text{ A}$$
$$I_d=2.3\times10^{-10}\text{ A}.$$

The value of the Raman-signal current I_A corresponds to an emission current from the photocathode of 6×10^{-17} A or 370 electrons/sec. With a room-temperature cathode sensitivity of 0.024 A/W at 6328 Å, this represents an incident light power of 2.5×10^{-15} W.

Using the value of I_d above, we may evaluate from (8) the minimum detectable signal current, $(I_A)_{\min}$, i.e., that corresponding to $S/N=1$. This gives $(I_A)_{\min}=1.4\times10^{-12}$ A. Hence, the cooled EMI 9558-A photomultiplier tube with our lock-in setup can detect the emission of 9 electrons/sec from the photocathode, pro-

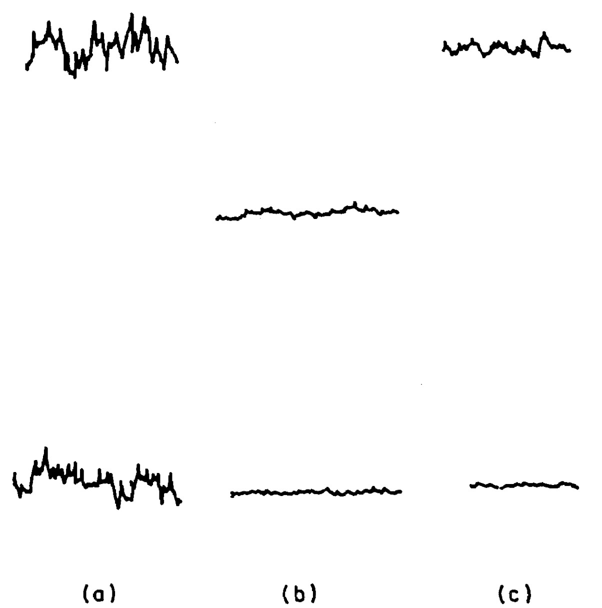

FIG. 2. Response of the lock-in, noise-voltage, and pulse-counting methods to the 992 cm^{-1} Raman band in benzene. Upper traces refer to observed signals with shutter open, lower traces refer to signal with shutter closed. (a) Lock-in; $S/N=8.2$, $R_L=1$ meg, $B_f=1/10$ cps, $G=5\times10^4$, $S=3.1$ V, $N=0.38$ V. (b) Noise-voltage; $S/N=27$, $R_L=1$ meg, $B_f=1/13$ cps, $G=1.5\times10^3$, $S=200$ mV, $N=7.5$ mV. (c) Pulse counting; $S/N=32$, $R_L=1$ meg, $\tau_0=10$ sec, $G=10^4$, $n=620$ cps, $n_d=40$ cps, $E=0.15$ V.

[2] *R. C. A. Phototubes and Photocells*, Technical Manual **PT-60**, 1963.

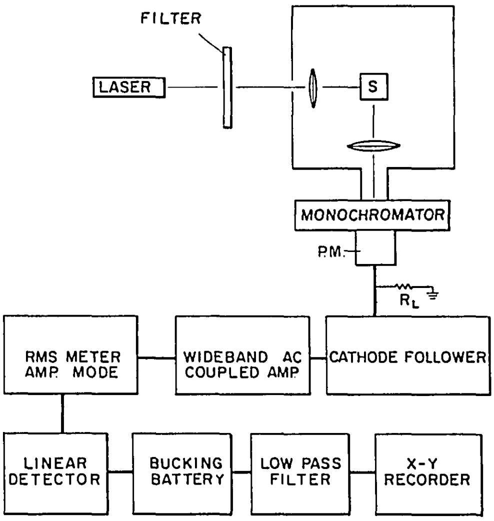

FIG. 3. Diagram of the noise-voltage technique for 90° Raman scattering.

duced by an incident light power of 5.7×10^{-17} W, or 177 photons/sec.

It should be noted that Eqs. (1) through (8) are equally applicable to dc detection, in which case B_f refers to the bandwidth of the dc amplifier used. Hence, with equivalent bandwidth, the same sensitivity, in principle, is obtainable by dc detection as by the lock-in technique. However, dc detection is limited by drift due to ohmic and light leakage and by the fact that dc amplifiers are less stable and more difficult to construct and operate than ac amplifiers. For these reasons no dc measurements were made.

NOISE-VOLTAGE DETECTION

A new method of detecting weak light signals was suggested by Yoh-Han Pao *et al.*[3,4] This method recovers the desired signal by detecting the associated shot noise.

It is well known that the output current from a photomultiplier tube irradiated by a photon flux consists of a dc component, which is usually referred to as the "signal," and an ac portion of unidirectional sharp pulses of variable amplitude, which is usually referred to as the "noise." These pulses are $\sim10^{-9}$ sec wide and arise from the emission of electrons from the cathode and dynodes multiplied by secondary emission. For low incident powers, there is more power in the ac portion than in the dc portion. Our measurements indicate that the S/N of this method is approximately three times as great as that of the lock-in technique.

The experimental setup is shown in Fig. 3. The signal from the photomultiplier, for weak light intensities, consists of a series of negative exponential voltage spikes. The small dc value of each spike is eliminated by ac coupling. The frequency components of the spikes above 1000 cps, which constitutes their major portion, are amplified and a linear detector removes their negligible positive portions. A low-pass filter extracts the dc component of the array of spikes which is passed to a recorder. The noise which appears at the output is due to the fluctuation of the current which appears within the pass-band of the final low-pass filter. The phototube dark current, the cathode follower noise, and the amplifier noise are processed by the system in the same way as the shot noise of interest. This contribution produces a shift of the dc base line of the recorded signal, which is bucked out by a battery.

We define V_1 and V_2 as the dc voltages of the signals at the output of the detector with the shutter open and closed, respectively. The observed signal, S, is the difference between the voltages measured at the output of the linear-detector–low-pass filter combination with the shutter opened and closed, respectively. The observed noise signal, N, is the average peak-to-peak value of the fluctuations at the output with the shutter open. Extending the results of Van der Ziel[1] to this case gives for the signal-to-noise ratio

$$\frac{S}{N}=(\sqrt{8})(V_1-V_2)/(B_fW_{01})^{\frac{1}{2}}, \quad (9)$$

where B_f is the output bandwidth, and W_{01} is the spectral power of the noise at zero frequency with the shutter open. For a circuit having a bandwidth B_s, the rms noise voltage for a square noise band is[1]

$$V_N=(W_{01}B_s)^{\frac{1}{2}}. \quad (10)$$

Substituting Eq. (10) into (9), we obtain

$$\frac{S}{N}=\frac{V_1-V_2}{V_N}\left(8\frac{B_s}{B_f}\right)^{\frac{1}{2}}. \quad (11)$$

From the nature of the signals previously described each voltage spike is produced by approximately μe coulombs of charge passing through the photomultiplier load, R_L. Hence, N spikes per second gives a spike voltage of $Ne\mu R_L$ volts at the photomultiplier output. Since the amplifier and detector do not appreciably change the shape of the spikes the average signal voltage for the shutter open is given by

$$V_1\simeq AGR_L(I_A+I_d), \quad (12)$$

while for the shutter closed it is

$$V_2\simeq AGR_LI_d, \quad (13)$$

and the noise voltage is

$$V_N\simeq AGR_L[2e\mu(I_A+I_d)B_s]^{\frac{1}{2}}. \quad (14)$$

A is the detector efficiency. We defined the quantities e, μ, I_A, and I_d previously for the lock-in case. Substituting Eqs. (12), (13), and (14) into Eq. (11) yields,

[3] Y. H. Pao, R. N. Zitter, and J. E. Griffiths, Bull. Am. Phys. Soc. **11**, 111 (1966).
[4] Y. H. Pao, R. N. Zitter, and J. E. Griffiths, J. Opt. Soc. Am. **56**, 1133 (1966).

for the signal-to-noise voltage ratio of the noise-voltage technique,

$$S/N \simeq [4I_A^2/e\mu(I_A+I_d)B_f]^{\frac{1}{2}}. \qquad (15)$$

The dependence of S/N on B_f and I_A, given by Eq. (15), was experimentally verified. The dependence on I_A was found to be linear, which is to be expected from Eq. (15) for $I_A < I_d$ and which was the case in our measurements. A typical experimental result is shown in Fig. 2(b).

For the reasons previously mentioned it is of interest to obtain the values of the signal current and dark current. They may be deduced from the experimental data of Fig. 2(b) using Eq. (15), and the relation between output signal voltage and photomultiplier current for a linear detector:

$$I_A \simeq S/AGR_L. \qquad (16)$$

From the data of Fig. 2(b), the output rms signal $S=0.2$ V, the peak-to-peak noise voltage is 0.0075 V which, as previously discussed, corresponds to an rms value of about 0.0015 V and hence a signal-to-noise ratio of about 135. Substituting these values for S and S/N into Eqs. (15) and (16), together with $A=0.84$ (measured), $\mu=10^6$, $G=1.5\times10^3$, $R_L=10^6\ \Omega$, $B_s=10^4$, and $B_f=1/13$ gives

$$I_A \simeq 16 \times 10^{-11}\ \mathrm{A}$$
$$I_d = 3.1\times10^{-10}\ \mathrm{A}.$$

Both I_A and I_d are in reasonable agreement with the values obtained for the lock-in method.

The minimum detectable signal current is approximately $1/\sqrt{2}$ of the lock-in minimum current.

ELECTRON-PULSE-COUNTING TECHNIQUE

A fourth method, though not new to x-ray and nuclear-particle detection, is the electron-pulse-height technique.[2,5,6] In this method, pulses in the anode circuit of the photomultiplier produced by photoelectrons from the cathode are counted by use of the system shown in Fig. 4. The pulses are amplified by a non-overload Hamner N-302 amplifier and passed to a single-channel pulse-height analyzer, which discriminates against pulses below a preset height, E, or against pulses outside the window between E and $E+\Delta E$. This window, ΔE, can be used to eliminate pulses arising from the ionization of gases in the tube, which are usually many orders of magnitude larger than the shot pulses. (Use of the window did not show any improvement in S/N.) Pulses above the threshold level produce pulses of equal height and duration, namely 30 V and $\sim$1 μsec, respectively. The array of pulses which constitute the signal can be displayed in the following ways; (1) the output pulses can be counted by a decade scalar which is set to count over a preset time determined by an electronic timer (1 to 10^4 sec), or (2) the output pulse

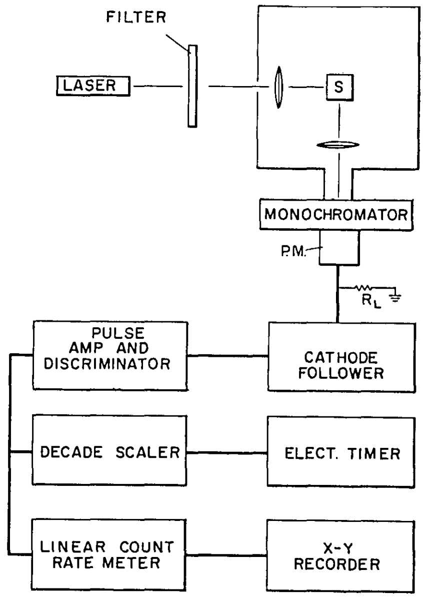

FIG. 4. Diagram of the pulse-counting technique for 90° Raman scattering.

rate (counts/sec) can be read by a linear count-rate meter which essentially extracts a dc level from the array of pulses with a time constant τ. The output of the rate meter is readily available at a variety of time constants ($\frac{1}{2}$ sec to 80 sec) and can conveniently be fed to 0–10 mV recorder.

The number of pulse heights per second greater than the discriminator bias, E, for the dark current and for the Raman 992 cm^{-1} line of benzene are shown in Fig. 5. Since the discriminator used did not allow operation with a bias less than 0.1 V, the form of the Raman signal distribution for very small bias is indicated by a dashed line.[6] The different characters of the distribution curves arise because the signal current originates at the cathode while the major part of the dark current comes from the dynodes.[2] Electrons originating at the dynodes result in smaller pulses because of the fewer stages of multiplication. The variation of the pulse height in the distributions is caused by the statistical variation of the secondary emission from the dynodes in the photomultiplier tube.

We can take advantage of the difference of the pulse-height distributions between the dark and signal count. By operating with the threshold level E near the peak of the $n_s(E)$ curve, indicated by the vertical line in Fig. 5(b), we maximize the signal to noise. To show this, it is necessary to discuss the signal-to-noise ratio of the electron pulse-height technique. We define the following parameters

τ = the time constant of the rate meter or the length of the counting period,

n = the number of counts/sec of signal plus dark current,

n_d = the number of counts/sec of dark current.

[5] E. H. Eberhardt, IEEE Trans. Nucl. Sci. NS-11(3), 48 (1964).

[6] R. F. Tusting, Q. A. Kerns, and H. K. Knudsen, IEEE Trans. Nucl. Sci. NS-9(3), 118 (1962).

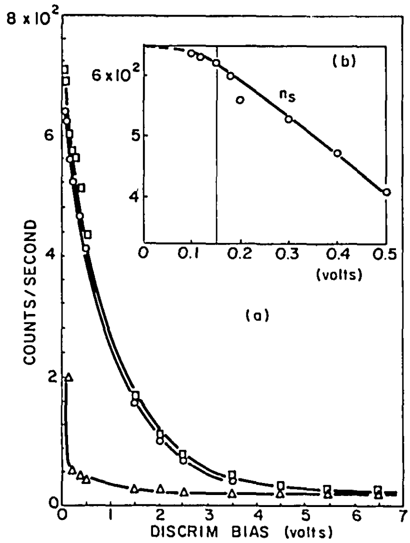

FIG. 5. (a) Distribution of pulse heights per sec above a discriminating bias voltage for a cooled EMI 9558 A photomultiplier operating at 1250 V; (b) Enlargement of the small pulse-height region, vertical line refers to pulse height for maximum signal-to-noise ratio. In (b), the dashed curve refers to expected behavior at low bias voltages which could not be observed. Δ Dark current, n_d; □ Dark and Raman signal count, $n=n_s+n_d$; ○ Raman signal, n_s; $\tau=10$ sec.

The signal count is the difference between the slit opened and closed electron-pulse count and can be written as

$$S=(n-n_d)\tau. \qquad (17)$$

The rms of the random fluctuations of the equal height pulses for a Poisson distribution, which is a very good approximation to a gaussian for large counts, is

$$N=(n\tau)^{\frac{1}{2}}. \qquad (18)$$

Hence, the S/N of the electron pulse-height technique using the count-rate meter is

$$S/N=(n-n_d)\tau/(n\tau)^{\frac{1}{2}}=n_s\tau/[(n_s+n_d)\tau]^{\frac{1}{2}} \qquad (19)$$

where n_s refers to the Raman-signal count rate. Differentation of Eq. (19) with respect to the discriminator bias, E, shows that the maximum value of S/N occurs when the slopes of the $n_s(E)$ and $n_d(E)$ curves are approximately equal. This occurs near $E=0.15$ V as indicated in Fig. 5(b) by the vertical line.

The dependence of S/N on n_s and τ, as given by Eq. (19), was experimentally verified. The minimum detectable signal count is found by setting $S/N=1$ in Eq. (19) which leads to $(n_s)_{\min}=1/(2\tau)[1+(1+4n_d\tau)^{\frac{1}{2}}]$. Substituting $\tau=10$ sec and $n_d=40$ counts/sec gives $(n_s)_{\min}=2$ counts/sec. The equivalent

$$(I_A)_{\min}=(n_s)_{\min}e\mu\sim 3.2\times10^{-13}\text{ A},$$

corresponding to 2 electrons/sec from the photocathode. This emission is produced by incident power of 1.3×10^{-17} W which corresponds to 40 photons/sec.

As previously mentioned, a comparison of the Raman anode dc currents for the three methods might be of interest. In this case the observed signal current is [for comparison we take the undiscriminated n_s value from Fig. 5(b)]:

$$(I_A)\simeq(n_s)_{\max}e\mu\simeq(650)(1.6\times10^{-19})(10^6) =10.4\times10^{-11}\text{ A}.$$

For this calculation, we assume that each anode pulse was produced by one electron being emitted from the cathode.[5,6] This value of anode current compares with values of 6×10^{-11} A and 16×10^{-11} A for the lock-in and noise-voltage techniques, respectively. The dark current of this method, using n_d from Fig. 5 is

$$I_d'\simeq n_d e\mu\simeq(40)(1.6\times10^{-19})(10^6)=6.4\times10^{-12}\text{ A},$$

which is considerably reduced below the previous methods by discrimination as can be surmised from Fig. 5(a).

SUMMARY

The results of our investigation of the capabilities of the detection techniques are summarized in Table I.

TABLE I. Theoretical and experimental results on detection of the 992 cm⁻¹ Raman band in benzene.

	Lock-in	Noise voltage	Electron pulse counting
$(S/N)_{\text{Theor.}}$	$\left(\dfrac{2I_A^2}{e\mu B_f(I_A+I_d)}\right)^{\frac{1}{2}}$	$\left(\dfrac{4I_A^2}{e\mu B_f(I_A+I_d)}\right)^{\frac{1}{2}}$	$n_s\tau/[(n_s+n_d)\tau]^{\frac{1}{2}}=I_A'/[e\mu B_f(I_A'+I_d')]^{\frac{1}{2}}$ [a]
$(S/N)_{\text{observed}}$ (peak-peak noise)	8.2	27	32
$(S/N)_{\text{observed}}$ (rms noise)	41	135	160
Observed signal	$I_A=6.0\times10^{-11}$ A	$I_A=16\times10^{-11}$ A	$n_s=650$ counts/sec; $I_A'=10.4\times10^{-11}$ A
Collected light power	2.5×10^{-15} W	6.7×10^{-15} W	4.3×10^{-15} W
Dark current or count	$I_d=2.3\times10^{-10}$ A	$I_d=3.1\times10^{-10}$ A	$n_d=40$ counts/sec; $I_d'=6.4\times10^{-12}$ A
Minimum detectable signal	1.4×10^{-12} A	1.0×10^{-12} A	2 counts/sec; 3.2×10^{-13} A
Minimum detectable light power	5.7×10^{-17} W	4×10^{-17} W	1.3×10^{-17} W

[a] Equivalent expression for S/N where $I_A'=n_se\mu$, $I_d'=n_de\mu$, and $B_f=1/\tau$.

For a signal of the intensity of the 992 cm^{-1} Raman band in benzene, the noise-voltage and electron-pulse counting methods give an effective S/N which are about three times as great as that of the lock-in method. The observed improvement of the noise-voltage method over that of the lock-in above that predicted theoretically is probably due to the ability of the ac coupled amplifier to reject the flicker $(1/f)$ component of noise below 1000 cps. With regard to the ultimate sensitivity of these techniques as characterized by the minimum detectable anode current, $(I_A)_{min}$, the pulse-counting scheme is more sensitive than the lock-in and noise-voltage methods.

The results of this work indicate that the pulse-counting and noise-voltage techniques are clearly superior to the lock-in and dc methods. The pulse-counting method is favored by a somewhat greater sensitivity and because this method lends itself most easily to an increase of the counting time $[\tau \sim (1/B_f)]$ and hence to an improvement of S/N.

In all of our measurements, the noise produced in the amplifiers and other electronic components was considerably less than that due to the dark current. The correctness of the formulae for S/N could not be unequivocally verified because of our inability to obtain independent measurements of the dark current. However, the following arguments can be given for their support: (a) The functional dependence on I_A and B_f were verified for all three methods experimentally studied; (b) from the observed signal and noise data the S/N formulas of the three techniques give similar values for I_A. In addition, the lock-in and noise-voltage formulas also yield similar values for I_d. These results are shown in Table I. Our results suggest a new technique combining the noise-voltage and pulse-counting methods, which could considerably improve both the S/N for weak signals $(I_A < I_d)$, and the minimum detectable signal current of the former. The new method would involve feeding the output of the discriminator wideband amplifier to a detector and low-pass filter combination as used in the noise-voltage setup. The analysis of this case proceeds in the same manner as for the noise-voltage method, but due to discrimination I_d will be reduced by a factor of 49 as shown in Table I. Since $S/N \propto I_A/I_d^{\frac{1}{2}}$ for $I_A < I_d$ and $(I_A)_{min} \propto I_d^{\frac{1}{2}}$, a reduction of I_d by a factor of 49 would improve S/N and $(I_A)_{min}$ by 7.

ACKNOWLEDGMENTS

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