Pulse Propagation in an Absorbing Medium

In a recent Letter, Chu and Wong[1] measure the envelope velocity of a picosecond laser pulse tuned to a strong absorption maxima of the A-exciton line in GaP:N. They state, on the basis of autocorrelation scans, that for $\Delta\nu_{\text{laser}} \ll \Delta\nu_{\text{abs}}$ no significant pulse distortion occurs. We wish to point out that a second-order autocorrelation scan gives no information as to pulse shape but can reveal changes in pulse width. It is insensitive to pulse asymmetries and will not reveal any frequency modulation. In fact the pulse shape must be separately known to determine the pulse half-width from an autocorrelation trace.[2] Figure 2 in Chu and Wong's Letter would indicate that some pulse reshaping and/or compression has taken place near the absorption peak.[3] The autocorrelation scans at -0.039 and $+0.020$ meV have full width at half maximum (FWHM) of 39 ps while those further off resonance have FWHM of 48 ps.

For a Gaussian pulse shape the autocorrelation of two pulses of widths a and b is a Gaussian of width $(a^2+b^2)^{1/2}$. This indicates that the on-resonance pulse was compressed from 34 to 19 ps.[4] For a double-sided exponential pulse shape their data imply an off-resonance pulse width of 18 ps and an on-resonance pulse width of 13 ps.

Single-pulse measurements using a streak camera, commercially available, with a resolution of 2 ps, will give direct measurements of pulse shape and velocity changes.

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Received 20 April 1982
PACS numbers: 42.10.-s, 03.50.De, 78.20.Dj

[1]S. Chu and S. Wong, Phys. Rev. Lett. 48, 738 (1982).
[2]E. Ippen and C. V. Shank, in *Ultrashort Light Pulses*, edited by S. L. Shapiro, Topics in Applied Physics Vol. 18 (Springer-Verlag, Berlin, 1977).
[3]M. D. Crisp, Phys. Rev. A 1, 1604 (1970).
[4]E. Ippen and C. V. Shank, in *Ultrashort Light Pulses*, edited by S. L. Shapiro, Topics in Applied Physics Vol. 18 (Springer-Verlag, Berlin, 1977), pp. 87–88; the ratio of autocorrelation FWHM to pulse FWHM is 1.41 for a Gaussian.

Chu and Wong Respond: We are grateful to Katz and Alfano[1] for pointing out the pulse compression in our data.[2] We will now show that the compression can be explained by the next-higher-order term in the Taylor-series expansion of the wave vector $k(\omega)$. Since the effect of pulse compression in the *linear* regime is not clearly identified in the literature, we present here a simplified version of Garrett and McCumber's[3] and Crisp's[4] analysis so that the physical significance of the Taylor-series expansion is obvious.

We model the absorption line at ω_0 as a Lorentzian,[4]

$$k=\frac{n_0\omega}{c}+\frac{i\alpha T_2}{1-i(\omega-\omega_0)T_2}$$

$$\simeq\frac{n_0\omega}{c}+i\alpha T_2[1+i\Delta\omega T_2-(\Delta\omega T_2)^2+\ldots], \quad (1)$$

where T_2 includes both inhomogeneous and homogeneous broadening of the line, and $\Delta\omega=\omega-\omega_0$. The Taylor-series expansion is valid when the laser bandwidth $\Delta\nu_L \ll T_2^{-1}$ and $\omega_L\simeq\omega_0$. Let $S(\omega)$ be the input pulse profile and use the Fourier-transform method[3] to get

$$\mathcal{E}(z,t)\sim\int_{-\infty}^{\infty}S(\omega)\exp[+i(kz-\omega t)]d\omega\sim\exp(-\alpha T_2 z)\int_{-\infty}^{\infty}S(\Delta\omega)\exp[-i\Delta\omega(t-n_0z/c+\alpha T_2^2 z)]$$

$$\times\exp(+\alpha\Delta\omega^2T_2^3z)d(\Delta\omega). \quad (2)$$

If we neglect the $\Delta\omega^2$ term and higher-order terms, inspection of (2) gives

$$\mathcal{E}(z,t)\sim\exp[-\alpha T_2 z\mathcal{E}(0,t'\equiv t-z/v_g)], \quad (3)$$

where $v_g\equiv d\omega/dk_r$ in the approximation of (1). Thus, the pulse propagates with the group velocity v_g even if $v_g>c$ or $\pm\infty$. The result is independent of the exact shape of $S(\Delta\omega)$ as long as $S(\Delta\omega)\sim 0$ when the approximation in (1) breaks down.

Since $\alpha\Delta\omega^2T_2^3z$ is positive, the additional term enhances the frequency wings of the laser pulse and this will result in a pulse compression. For example, a Gaussian pulse with $S(\Delta\omega)\sim\exp(-\Delta\omega^2/2t^2)$ yields

$$\mathcal{E}(z,t)\sim\exp(-t'^2/2\Delta^2),$$

$$\Delta^2\equiv 2(\tau^2/2-\alpha T_2^3z) \quad (4)$$

and t' is given by (3). Thus, the emerging pulse is a Gaussian compressed by a factor $(1-2\alpha T_2^3z/t^2)^{1/2}$ *but still traveling at the group velocity.* A 34-ps Gaussian pulse with a 48-ps autocorrelation width (full width at half maximum) propagating through our $[N]=1.5\times10^{17}$ cm^{-3} sample ($\Delta\nu_{abs}=0.16$ meV, $\alpha=5.7$) gives a cross-correlation pulse of 40 ps as observed. In our other samples our experimental results are also quantitatively consistent with (4).

As long as $\Delta\omega^3$ terms or higher can be neglected, pulse propagation still occurs at the group velocity.

Although the theoretical analysis is most easily done with Gaussian pulses, our experimental results, which were obtained with non-Gaussian pulses,[2] show that the pulse behavior does not depend strongly on the exact shape of the pulse provided that $\Delta\nu_L\ll\Delta\nu_{abs}$ and z is small. We feel that both numerical analysis and further experimental work will clarify these details.

We are grateful to S. L. McCall and M. D. Sturge for helpful discussions.

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Received 21 May 1982
PACS numbers: 42.10.-s, 03.50.De, 78.20.Dj

[1]A. Katz and R. R. Alfano, preceding Comment [Phys. Rev. Lett. 49, 1292 (1982)].
[2]S. Chu and S. Wong, Phys. Rev. Lett. 48, 738 (1982).
[3]C. G. B. Garrett and D. E. McCumber, Phys. Rev. A 1, 305 (1970).
[4]M. D. Crisp, Phys. Rev. A 4, 2104 (1971), and 1, 1604 (1970).