SECOND HARMONIC GENERATION IN BIREFRINGENT OPTICAL FIBERS

Jamal T. MANASSAH, R.R. ALFANO and Samir A. AHMED

Photonics Engineering Laboratories, Department of Electrical Engineering, City College of New York, Convent Avenue at 140th Street, New York, NY 10031, USA

Received 19 November 1985; accepted for publication 21 January 1986

Second harmonic generation in optical fibers is investigated. A scheme to combine material, modal and geometrical dispersion to achieve phase matching is proposed. Conversion efficiency, as a function of the fiber's parameters and the incoming electric field strength, is computed.

Low-loss fibers are an excellent transmission medium for optical communications networks [1]. Optical fibers are also excellent nonlinear materials, pulse compression [2] to 8 fs using self-phase modulation being so far the most noted achievement. The low efficiency of the coupling between lasers and optical fibers and the need for signal amplification in long-haul communication systems prompted studies of active fibers as quantum amplifiers [3]. Fibers are also candidates for some optical computation schemes. Fibers photonic integrated systems are becoming serious contenders in many applications. In this letter, we add one more possible application. We investigate second harmonic generation [4] in fibers due to quadrupolar terms [5].

The fiber's ability to trap electric field energy densities over long distances permits large conversion into second harmonics for a relatively low value of the nonlinearity. The phase matching between the propagation constants at ω and 2ω ($k^{(2\omega)} \sim 2k^{(\omega)}$) can be adjusted if necessary through modal dispersion. The geometrical anisotropy in elliptical fibers [6] produces birefringence, which adds to the material and modal dispersion. This birefringence can potentially be used to improve phase matching and consequently optimize second harmonic conversion efficiency.

The wave equation for a linear step index waveguide is given by

$$[\nabla_T^2 - \beta_m^2 + (n^2/c^2)\omega^2]\ \phi_m(r) = 0\ , \tag{1}$$

where the electric field is given by

$$E(r,z,t) = \psi = \phi(r)\,e^{-i\beta z}\,e^{i\omega t}\ . \tag{2}$$

The fundamental solution of eq. (1) is given by [7]

$$\phi_m(r) = J_0(l_m r/R)\ , \tag{3a}$$

where

$$\beta_m^2 = n^2\omega^2/c^2 - (l_m/R)^2\ . \tag{3b}$$

The l_m is the mth root of the equation

$$\frac{J_1(x)}{xJ_0(x)} + \frac{1-2\Delta}{(2\Delta n^2\omega^2R^2/c^2 - x^2)^{1/2}} \times \frac{K_1((2\Delta n^2\omega^2R^2/c^2 - x^2)^{1/2})}{K_0((2\Delta n^2\omega^2R^2/c^2 - x^2)^{1/2})} = 0\ , \tag{4}$$

where R is the core radius and the square of the index of refraction of the cladding is $n^2(1-2\Delta)$. Considering that the lowest root l_1 corresponds to the most dominant mode, we obtain

$$\phi_1(r) = J_0(l_1 r/R)\ , \tag{5a}$$

and

$$\beta_1^2 = n^2\omega^2/c^2 - (l_1/R)^2\ . \tag{5b}$$

The subscript 1 will henceforth be omitted. Both β and l are functions of ω explicitly and implicitly through the dependence of n on ω.

In the two modes problem (ω and 2ω), to first or-

0.375-9601/86/$ 03.50 © Elsevier Science Publishers B.V. (North-Holland Physics Publishing Division)

der we shall treat the r dependence in a linear fashion and use separation of variables for the transverse and longitudinal components. The ith component of the nonlinear polarization, in the dipolar approximation, at $\omega_1 = \omega_3 - \omega_2$ is given by

$$(P_{\mathrm{NL}}^{(1)})_i = d_{ijk} E_j^{(3)} (E_k^{(2)})^* \exp[\mathrm{i}(\omega_3 - \omega_2)t] , \tag{6}$$

where $E_j^{(s)}$ is the j-component of the spatial part of the electric field with frequency ω_s and d_{ijk} is the nonlinear coefficient. However, in isotropic media, the average value of d_{ijk}, which is obtained from the dipolar term in the multipole field expansion, is zero. Therefore, the leading contribution to P_{NL} will come from the quadrupole term of the charge distribution (permanent and/or induced) multipole field expansion, i.e., the term with the quadrupole moment multiplied by the gradient of the electric field. Specifically,

$$(P_{\mathrm{NL}}^{(1)})_i = q_{ijkl} E_j^{(3)} (\nabla_k E_l^{(2)})^* \exp[\mathrm{i}(\omega_3 - \omega_2)t] . \tag{7}$$

If the magnitude of the gradient of the electric field is approximated by $2\pi E/\lambda$, i.e., the derivative of the envelope is neglected, the effective coupling constant is $q \times 2\pi/\lambda$. If q and p are the magnitudes, respectively, of the quadrupole and the dipole moments, the effective-coupling constant can be approximated by $(2\pi/\lambda)(q/p)d$, i.e., the magnitude of the effective coupling is down by a factor $2\pi a/\lambda$ from d, where a is the linear dimension of the microsystem (atoms, molecules, etc.). This factor is approximately 10^{-3} for optical frequencies.

Approximating the gradient by the z-derivative and denoting $(q\mu_0\omega_i^2)$ by q_i', then the coupled equations for the electric fields of the primary and second harmonic are given by

$$[\nabla_{\mathrm{T}}^2 + (\partial^2/\partial z^2) + (n^{\omega}\omega/c)^2]\, \phi^{(\omega)}(r) u_1(z) = q_1' \phi^{(\omega)}(r) \phi^{(2\omega)}(r) u_2(z)\, \partial u_1(z)/\partial z , \tag{8}$$

$$[\nabla_{\mathrm{T}}^2 + (\partial^2/\partial z^2) + (n^{2\omega} 2\omega/c)^2]\, \phi^{(2\omega)}(r) u_2(z) = q_2' \phi^{(\omega)}(r) \phi^{(2\omega)}(r) u_1(z)\, \partial u_1(z)/\partial z , \tag{9}$$

where ∇_{T}^2 is the transverse portion of the laplacian. Using (1) in (8) and (9), we obtain

$$[(\partial^2/\partial z^2) + (\beta^{(\omega)})^2]\, \phi^{(\omega)}(r)\, u_1(z) = q_1' \phi^{(\omega)}(r)\, \phi^{(2\omega)}(r)\, u_2(z)\, \partial u_1(z)/\partial z , \tag{10}$$

$$[(\partial^2/\partial z^2) + (\beta^{(2\omega)})^2]\, \phi^{(2\omega)}(r)\, u_2(z) = q_2' \phi^{(\omega)}(r)\, \phi^{(2\omega)}(r)\, u_1(z)\, \partial u_1(z)/\partial z , \tag{11}$$

where

$$\beta^{(\omega)} = (n^{(\omega)}\omega/c)^2 - (l^{(\omega)}/R)^2 . \tag{12}$$

Eqs. (10), (11) are averaged over the cross section [8], respectively, with the weight functions $\phi^{(\omega)}(r)$ and $\phi^{(2\omega)}(r)$. Defining γ_1 and γ_2 as

$$\gamma_1 = \left(\int_0^1 J_0^2(l^{(\omega)}x) J_0(l^{(2\omega)}x) x\, \mathrm{d}x \right) \times \left(\int_0^1 J_0^2(l^{(2\omega)}x) x\, \mathrm{d}x \right)^{-1} , \tag{13}$$

$$\gamma_2 = \left(\int_0^1 J_0^2(l^{(\omega)}x) J_0(l^{(2\omega)}x) x\, \mathrm{d}x \right) \times \left(\int_0^1 J_0^2(l^{(2\omega)}x) x\, \mathrm{d}x \right)^{-1} \tag{14}$$

eqs. (10), (11) reduce then to

$$[\partial^2/\partial z^2 + (\beta^{(\omega)})^2]\, u_1(z) = c_1 u_2(z)\, \partial u_1(z)/\partial z , \tag{15}$$

$$[\partial^2/\partial z^2 + (\beta^{(2\omega)})^2]\, u_2(z) = c_2 u_1(z)\, \partial u_1(z)/\partial z , \tag{16}$$

where $c_i = q_i'\gamma_i$ represents the average coupling coefficients over the field distribution along the radius.

To solve eqs. (15), (16), we will use the method of multiple scales [9] which is specially suited for nonlinear differential equations of the above gyroscopic type. Let the solutions be written as

$$u_1 = \epsilon u_{11}(Z_0, Z_1) + \epsilon^2 u_{12}(Z_0, Z_1) + \ldots , \tag{17}$$

$$u_2 = \epsilon u_{21}(Z_0, Z_1) + \epsilon^2 u_{22}(Z_0, Z_1) + \ldots , \tag{18}$$

where $Z_0 = z$, $Z_1 = \epsilon z$ etc. To order ϵ, the differential equations are

$$D_0^2 u_{11} + (\beta^{(\omega)})^2 u_{11} = 0 , \tag{19a}$$

$$D_0^2 u_{21} + (\beta^{(2\omega)})^2 u_{21} = 0 , \tag{19b}$$

where $D_0 = \partial/\partial Z_0$. The solutions of (19) are given by

$$u_{11} = A_1(Z_1) \exp(\mathrm{i}\beta^{(\omega)} Z_0) + \mathrm{c.c.} , \tag{20a}$$

$$u_{21} = A_2(Z_1) \exp(\mathrm{i}\beta^{(2\omega)} Z_0) + \mathrm{c.c.} \tag{20b}$$

The differential equations, obtained from order ϵ^2, are

$$D_0^2 u_{12} + 2D_0 D_1 u_{11} + (\beta^{(\omega)})^2 u_{12} = c_1 (D_0 u_{12}) u_{21} , \tag{21a}$$

$$D_0^2 u_{22} + 2D_0 D_1 u_{21} + (\beta^{(2\omega)})^2 u_{22} = c_2 u_{11} (D_0 u_{11}) . \tag{21b}$$

The secular conditions from these equations reduce to

$$2\beta^{(\omega)} A_1' + \beta^{(\omega)} c_1 A_2 A_1^* \exp(\mathrm{i}\sigma Z_1) = 0 , \tag{22}$$

$$-2\beta^{(2\omega)} A_2' + \beta^{(\omega)} c_2 A_1^2 \exp(-\mathrm{i}\sigma Z_1) = 0 , \tag{23}$$

where prime is differentiation with respect to Z_1 and the detuning factor σ is given by

$$\beta^{(2\omega)} = 2\beta^{(\omega)} + \epsilon\sigma . \tag{24}$$

Denoting the amplitude and phase of A_i by a_i and θ_i,

$$A_i = \tfrac{1}{2} a_i \exp(\mathrm{i}\theta_i) + \text{c.c.} , \tag{25}$$

eqs. (22), (23) reduce to

$$a_1' = -\tfrac{1}{4} c_1 a_1 a_2 \cos\gamma , \tag{26}$$

$$a_2' = (\beta^{(\omega)} c_2 / 4\beta^{(2\omega)}) a_1^2 \cos\gamma , \tag{27}$$

$$\theta_1' = -\tfrac{1}{4} c_1 a_2 \sin\gamma , \tag{28}$$

$$\theta_2' = -(\beta^{(\omega)} c_2 / 4\beta^{(2\omega)} a_2) a_1^2 \sin\gamma , \tag{29}$$

where

$$\gamma = \theta_2 - 2\theta_1 + \sigma Z_1 . \tag{30}$$

The integrals of motion for the above set of equations are

$$a_1^2 + \upsilon a_2^2 = L_1 , \tag{31}$$

where

$$\upsilon = c_1 \beta^{(2\omega)} / c_2 \beta^{(\omega)} \tag{32}$$

and

$$a_2 a_1^2 \sin\gamma - (2\upsilon\sigma/c_1) a_2^2 = L_2 . \tag{33}$$

For the initial conditions $a_1^2(0) = \mathcal{E}$ and $a_2(0) = 0$, $L_1 = \mathcal{E}$ and $L_2 = 0$. Parametrizing a_1^2 as

$$a_1^2 = \zeta , \tag{34}$$

the equation of motion for ζ is then given by

$$\left(\frac{\mathrm{d}\zeta}{\mathrm{d}z}\right)^2 = \frac{c_1 \mathcal{E}}{4\upsilon} \left[\left(\zeta^2 - \frac{4\upsilon\sigma^2}{c_1^2 \mathcal{E}} (1-\zeta)\right)(1-\zeta)\right] ; \tag{35}$$

its solution is

$$Kz = F(\chi, \eta) , \tag{36}$$

where

$$\zeta_3 - \zeta = (\zeta_3 - \zeta_2) \sin\chi , \tag{37}$$

$$\eta = [(\zeta_3 - \zeta_2)/(\zeta_3 - \zeta_1)]^{1/2} , \tag{38}$$

$$K = \tfrac{1}{4} c_1 [\mathcal{E} (\zeta_3 - \zeta_1)/\upsilon]^{1/2} . \tag{39}$$

F is the elliptic integral of the first kind, and $\zeta_1 \leqslant \zeta_2 \leqslant \zeta_3 = 1$ are the roots of the cubic equation inside the brackets on the right hand side of eq. (35). The minimum of ζ is ζ_2. ζ_2 is zero when σ, the detuning, is zero, i.e., the energy can be completely transferred to the second harmonic mode if we have perfect phase matching ($\beta^{(2\omega)} = 2\beta^{(\omega)}$). In case that $\beta^{(2\omega)} \neq 2\beta^{(\omega)}$ for the lowest mode, the detuning σ can be zero for a value of $l^{(2\omega)}$ which is not the lowest root of eq. (4). The distance at which ζ reaches its first minimum is given by

$$z = (1/K)\, \mathsf{K}(\zeta_3 - \zeta_2)/(\zeta_3 - \zeta_1) , \tag{40a}$$

where

$$\mathsf{K}(m) = \int_0^{\pi/2} (1 - m \sin^2\theta)^{-1/2}\, \mathrm{d}\theta , \tag{40b}$$

i.e., the complete elliptic integral of the first kind. This distance is inversely proportional to the nonlinear coupling constant and the amplitude of the incoming electric field.

Next, we propose a scheme to fine tune the phase matching if the material and modal dispersions do not balance for particular modes. Elliptical deformation of the fiber core causes birefringence [10]. These deformed fibers have been used extensively in polarization state applications. Slight elliptical deformation in the core cross-section can be computed through perturbation theory. In this case, the perturbation is applied to the boundary conditions. By means of a coordinate transformation, the new boundary conditions can be transformed back to their unperturbed form, however, this transformation would correspondingly change the functional expression for the differential operators. These changes in the transverse laplacian are then considered as the perturbation operator. The correction to the propagation constant is then obtained by standard perturbation techniques for computing corrections to an eigenvalue. Specifically, if the equation for the circular cross section is

$$x^2 + y^2 - R^2 = 0\,, \tag{41}$$

and the equation for the ellipse is

$$x^2/R_1^2 + y^2/R_2^2 - 1 = 0\,, \tag{42}$$

then the new variables transformation is given by

$$x = R_1 x'/R\,, \quad y = R_2 y'/R\,. \tag{43}$$

The transverse laplacian operator in the new coordinates is then given by

$$\nabla_T^2 = (R^2/R_1^2)\,\partial^2/\partial x'^2 + (R^2/R_2^2)\,\partial^2/\partial y'^2\,, \tag{44}$$

and the perturbation operator takes the form:

$$H' = (R^2/R_1^2 - 1)\,\partial^2/\partial x'^2 + (R^2/R_2^2 - 1)\,\partial^2/\partial y'^2\,, \tag{45}$$

which will be small provided that R_1 and R_2 are close to R. In an elliptical deformation that conserves the cross-sectional area, and for small ellipticity:

$$R_1 = R(1 + \tfrac{1}{2}e)\,, \quad R_2 = R(1 - \tfrac{1}{2}e)\,, \tag{46}$$

where e is the ellipticity. The perturbation operator is then given by

$$H' = e(-\,\partial^2/\partial x'^2 + \partial^2/\partial y'^2)\,, \tag{47}$$

and the first order correction to β^2, where β^2 is defined in eq. (1), is given by

$$\delta\beta^2 = \frac{\int \phi(x', y')H'\,\phi(x', y')\,\mathrm{d}x'\,\mathrm{d}y'}{\int \phi^2(x', y')\,\mathrm{d}x'\,\mathrm{d}y'}\,. \tag{48}$$

The geometrical birefringence defined by

$$B = \tfrac{1}{2}(\beta_x - \beta_y)\,\lambda \tag{49}$$

was computed by many authors [10] to be given for a step index fiber by

$$B = e\,n\,\Delta\,G(\nu)\,, \tag{50}$$

where β_x and β_y are the propagation constants for x and y polarization, λ is the wavelength of light in vacuum, ν is the normalized frequency defined as

$$\nu = (2\pi/\lambda)\,n\,R\,\sqrt{2\Delta} \tag{51}$$

and $G(\nu)$ is a function of ν [11]. In the range of interest, $\nu \sim 1$, $G(\nu)$ is in the range 0.3 to 0.6. The computation of geometrical birefringence for large eccentricities has also been reported in ref. [11].

In the above calculations we outlined two techniques for phase matching, modal dispersion and geometrical birefringence. Other techniques for reducing the detuning include inducing birefringence by mechanical means such as stress and twist [12]. The finesse of the required tuning is proportional to the magnitude of the incoming electric field. For a 10 mW cw beam, tuning should be to within 1 part per ten billion for efficient conversion. The corresponding distance for this optimal conversion is 10^3 m. Manufacturing-wise this implies producing a 1 km fiber with an ellipticity controlled to one part per million. However, the typical source width (1 Å) relaxes this condition to a required tolerance of one part per hundred.

References

[1] D.G. Thomas et al., in: Innovations in telecommunications, ed. J. Manassah, part A (Academic Press, New York, 1982) p. 437.
[2] W. Knox, R. Fork, M. Downer, R. Stolen and C. Shank, Appl. Phys. Lett. 46 (1985) 1120.
[3] M. Dzhibladze, E. Teplitskii and R. Erikashvili, Sov. J. Quantum Electron. 11 (1984) 132.
[4] P. Franken, A. Hill, C. Peters and W. Weinrich, Phys. Rev. Lett. 7 (1961) 118.
[5] S. Kielich, Acta Phys. Pol. 33 (1968) 89; IEEE J. Quantum Electron. QE-4 (1968) 744.
[6] C. Yeh, J. Appl. Phys. 33 (1962) 3235.
[7] N. Tzoar and M. Jain, in: Fiber optics, eds. B. Bendow and S. Mitra (Plenum, New York, 1979) p. 313.
[8] A. Hasegawa and Y. Kodama, Proc. IEEE 69 (1981) 1145.
[9] A.H. Nayfeh, Introduction to perturbation techniques (Wiley, New York, 1981).
[10] D. Tjaden, Philips J. Res. 33 (1978) 254; R. Sammut, Electron. Lett. 16 (1980) 728.
[11] K. Okamoto, T. Hosaka and Y. Sasaki, IEEE J. Quantum Electron. QE-18 (1982) 496.
[12] D. Payne, A. Barlow and J. Ramskov Hansen, IEEE J. Quantum Electron. QE-18 (1982) 477.