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Demonstration of a picosecond optical-phase-conjugation-based residue-arithmetic computation

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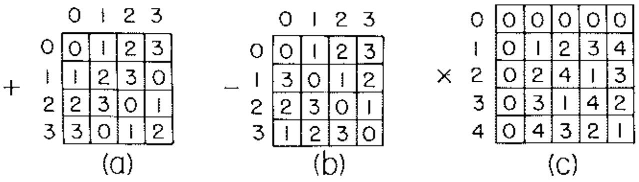
The use of the optical phase-conjugation (OPC) process for optical residue computation is proposed. By using an OPC-based parallel switching array, various optical position-coded residue-mapping units for carry-free addition, subtraction, and multiplication operations are described. Experimental results obtained with a picosecond mode-locked Nd^{3+}:YAG laser are presented to support the proposal.

A residue number system has been suggested, because of its accurate parallel-processing capability, for carry-free arithmetic, such as addition, subtraction, and multiplication, operations.[1] An electro-optical (E-O) liquid-crystal spatial light modulator has been used to obtain the required residue-processing cyclic response.[2] E-O position-encoded residue mapping units have been described that use either an optical waveguide coupler array or a laser-diode grid.[3–6] Other alternatives, such as the holographic optical truth-table look-up, optical symbolic substitution, and optical second-harmonic-generation residue-processing techniques, have been described.[7–10] In addition, various optical decimal-to-residue and residue-to-decimal conversion schemes are available.[11,12] Optical phase conjugation (OPC) by degenerate four-wave mixing is a technique that reverses an input beam's phase and propagation directions. OPC has found many applications in optical signal and image processing.[13] By using an ultrafast optical χ^3 nonlinear material, picosecond OPC switching has been demonstrated.[14] Recently various OPC-based digital optical computing elements have been proposed and demonstrated that use its speed and parallel-processing advantages.[15–19] In this Letter an OPC device together with a spatial-position encoding scheme is described for an optical residue processor (ORP). A prototype residue-addition mapping element that consists of an OPC cell and a combination of mirror, a beam splitter, and a cylindrical lens is described. The use of this add element for optical residue subtraction and multiplication by rearranging the input and output channels is also described. Some preliminary experimental results are presented to demonstrate an ORP that uses a picosecond mode-locked Nd^{3+}:YAG laser as the source.

The core of an ORP is a set of prime modulo residue-mapping elements.[3] For an ORP the two position-coded integers are directed to and switched in parallel by these mapping elements. The ORP's dynamic range is the product of all the prime moduli units employed. As an example, truth tables for a mod 4 residue addition (subtraction) and a mod 5 residue multiplication are presented in Fig. 1. For addition (subtraction) [see Figs. 1(a) and 1(b)], left (right) cyclic rotation is needed. It has been noted[4] that, by rearranging the input and output channels, one can use a residue-add unit to perform residue subtraction and multiplication operations. Thus, for any prime modulo residue arithmetic, a residue adder is the fundamental element. To implement a residue-add truth table, an optical switching array may be employed. The residue integers are encoded as spatial positions. For example, for the mod 4 addition, the four addend (summand) numbers 0, 1, 2, and 3 are encoded into four insecting horizontal (vertical) channels. For each of the 16 possible addition pairs, only one intersection will be addressed. To generate the add output, the 16 addressable outputs need to be grouped into 4 sum channels. Because of the cyclic shift property of residue addition [see Fig. 1(a)], except for the four output channel 3 light spots that appear along the off-main diagonal axis, the other spots, i.e., for output channels 0, 1, and 2, appear on both sides of the off-main diagonal axis and are separated by a fixed spatial constant along this off-main diagonal direction. Thus, for the output grouping, conventional shift-invariant optical elements can be used.

In Fig. 2, a schematic of an OPC mod 4 residue adder is shown. In the top part of the figure, an OPC cell with three input beams, two counterpropagating and one off axis, is depicted. The three input beams interact within the OPC material and generate an output



+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

(a)

−	0	1	2	3
0	0	1	2	3
1	3	0	1	2
2	2	3	0	1
3	1	2	3	0

(b)

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

(c)

Fig. 1. Truth tables for mod 4 residue addition (a) and subtraction (b) and for mod 5 residue multiplication (c). After certain input and/or output permutation, both (b) and (c) can be expressed in terms of (a).

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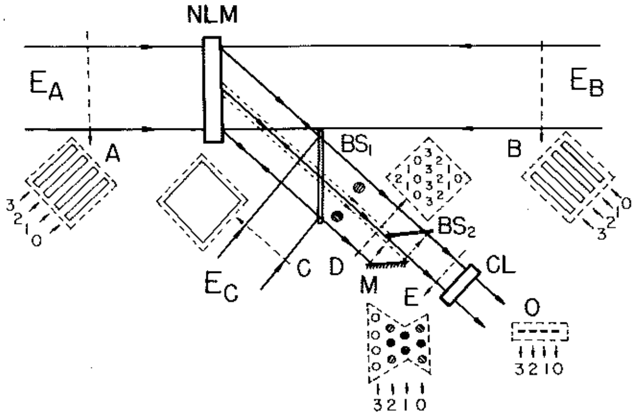

Fig. 2. A schematic of an OPC-based spatial position-coded mod 4 optical-residue addition unit. E_A, E_B, and E_C, three input beams; BS's, beam splitters; M, mirror; CL, cylindrical lens; NLM, nonlinear material. A, B, C, D, E, and O are beam profiles at various locations.

that counterpropagates with respect to the off-axis input beam. For our purpose, the two counterpropagating beams carry the two position-coded (see beam profiles A and B of Fig. 2) mod 4 inputs. The third input beam serves as an optical power supply (see beam profile C). The OPC output, which may contain as many as 16 light spots (see beam profile D), is directed by a beam splitter to a postprocessing (output grouping) unit. To group the outputs, a beam combiner that consists of a mirror–beam-splitter pair oriented at a 45° direction is inserted into the beam. This beam combiner is tuned to provide a gap that allows the four diagonal-output channel 3 spots to pass while all the other spots are properly grouped along the diagonal direction (see beam profiles D and E in Fig. 2). To generate the final addition mapping output, a cylindrical lens that spatially integrates the results along the off-main-diagonal direction (see beam profile O) is employed.

The device described above can also be used for residue-subtraction mapping. Let the additive inverse $\bar{A}$ of a residue number A be defined as[4] $|A + \bar{A}|_N = 0$ (mod N). To subtract a residue number A, the addition of the inverse $\bar{A}$ is performed. Thus, for example in the mod 4 case, to convert addition to subtraction the switching from the addend channel 1 (3) to the subtrahend channel 3 (1) needs to be performed. For mod N multiplication, where N is prime, a mod $N - 1$ add unit can be employed. When a homomorphic approach is used,[4] in addition to the input (a loglike) conversion the output channels also need be permuted (an inverse loglike conversion). For example, for a mod 5 multiplication, an exchange between channels 2 and 3 for both the input and the output needs to be incorporated before a mod 4 addition element can be used.

A 32-psec pulse from a Quantel mode-locked Nd^{3+}:YAG laser is used to demonstrate the basic principles of an ORP. The experiment was divided into two parts: (1) to implement the OPC switching array and (2) to construct a mirror–beam-splitter coupler for residue-addition output grouping. First, the laser output beam was magnified to a 1.5 cm × 1.5 cm area from which only the central portion (about 1 cm^2) was selected for the experiments. The OPC nonlinear material was a commercially available 5.08 cm × 5.08 cm, 2-mm-thick 3–68 Corning glass filter. We understand that this filter consists of microcrystal structures of CdS_xSe_{1-x} ternary compound semiconductors embedded in an amorphous glass matrix.[20] In addition to being fast (50 psec) (Ref. 21) and of relatively high optical nonlinearity [$\chi^{(3)} \approx 10^{-28}$–$10^{-27}$ MKS units], the medium is isotropic. For the input position encoding, although one can use diode lasers and cylindrical lenses to incorporate a real-time point-to-bar shape-modulation scheme, in our preliminary experiment absorption masks were employed. Each mask slot was 1 cm long and 1.5 mm wide. To prevent spatial channel cross talk, for each mask and between every two consecutive light channels an identical-sized guard band was used. In Fig. 3, the experimental result of a 4 × 4 OPC switching array is shown. Two input groups (four channels each) of crossed light bars were used; the 16 light dots generated had approximately equal intensity. For a mod 4 residue adder, these outputs are then guided to a mirror–beam-splitter beam combiner. In our experiment, the beam combiner was mounted on an adjustable translation stage. The equal-splitting-ratio antireflectively coated beam splitter was 1 mm thick. The combiner was first adjusted to create a 2-mm-wide gap to allow for the passage of the four main diagonal light spots. It was then aligned to be parallel to a 45° direction and adjusted so that for the 16 input spots the output light pattern appeared to be similar to beam pattern E of Fig. 2. Finally, for one-dimensional spatial integration, a cylindrical lens was employed. With this alignment, a system verification was performed (see Fig. 4). First, for each of the four possible mod 4 addition outputs, four properly positioned light dots were obtained (see the top row of Fig. 4). The light patterns generated by using the beam combiner are shown in the middle row of Fig. 4. Note that, except for output channel 3, each of the output channels has been rearranged so that it can be located along a vertical line. Finally, in the bottom row of Fig. 4, the recorded mod 4 addition patterns made using a cylindrical lens are shown. Based on this implementation concept, other

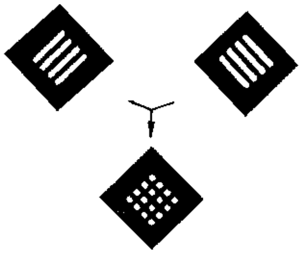

Fig. 3. Experimental result of OPC input intensity modulation. The intensity modulation (AND operation) of two input bar arrays results in a 16-light-dot output pattern that is used to generate the content of a mod 4 residue-addition truth table.

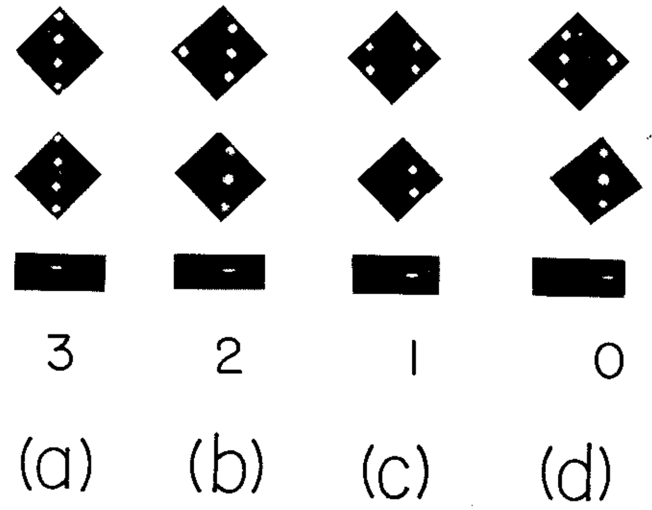

Fig. 4. Experimental results of the OPC mod 4 adder. The light patterns in the first row (corresponding to beam profile D of Fig. 2) show the four truth-table output locations for the addition results 0, 1, 2 and 3, respectively. The patterns in the second row (corresponding to Fig. 2 beam profile E) represent the corresponding beam combiner's outputs. The last row's patterns, which correspond to output O of Fig. 2, show the spatially integrated results.

prime modulo add units can also be realized. By using a set of parallel different modulo OPC add units together with some input and output conversion and permutation devices, an OPC-based ORP can be constructed. To reduce space–bandwidth-product problems caused by a large set of parallel mapping units, the binary coded residue (BCR) technique may be helpful. It can be shown that the dynamic range of a set of sixteen 5-bit BCR processing elements can be as high as 10^{23}. With the material and laser source employed it is difficult to achieve a high repetition rate. However, by using other high $\chi^{(3)}$ nonlinear materials with absorption peaks not in the operational range, the heating problem can be circumvented, leading to OPC-based high-repetition-rate residue processing.

The major advantages of this OPC-based ORP method are the following:

(1) It uses the degenerate wave mixing effect so that the input and output of the processor are identical-frequency optical rather than E-O hybrid signals.

(2) The use of OPC allows for an ultrafast processing speed. Although in our proof-of-principle experiments a 50-psec material was used, much faster OPC materials, such as nonlinear polymers, are available.[14] Thus, compared with other existing residue-mapping schemes, this OPC-based method offers a speed advantage.

(3) The overall system uses simple optical elements, such as mirrors and beam splitters, allowing it to be miniaturized to an integrated-optics scale. With some input and output optical waveguides, the addition-mapping element can also be converted to perform subtraction and multiplication mapping. Thus an OPC-based compact, ultrafast ORP can be constructed.

In summary, an OPC-based spatial-position-encoded ORP technique has been proposed and demonstrated. To implement a particular modulo residue-addition mapping element, an OPC setup is employed. To group the truth-table contents, a compact mirror–beam-splitter beam combiner was used. Experimental verification of a mod 4 residue-addition unit using a picosecond mode-locked Nd^{3+}:YAG laser as the source and a semiconductor-doped glass nonlinear material was described. Extension of the technique to residue subtraction and multiplication mapping was also discussed.

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