Reprinted from Optics Letters, Vol. 13, page 1090, December 1988.

Self-focusing, self-phase modulation, and diffraction in bulk homogeneous material

Jamal T. Manassah, P. L. Baldeck, and R. R. Alfano

Department of Electrical Engineering, The City College of New York, New York, New York 10031

Received May 25, 1988; accepted September 20, 1988

The simultaneous effects of self-focusing, self-phase modulation, and diffraction on the propagation of an ultrafast pulse in a homogeneous Kerr medium are described. The competing effects of self-focusing and diffraction are shown to modify the shape and magnitude of the pulse amplitude and phase. These modifications are shown to affect the spectral distribution of the supercontinuum from that predicted by the conventional self-phase-modulation theory.

Self-focusing and self-phase modulation as manifestations of the Kerr nonlinearity have been the subject of intense theoretical and experimental studies since the seminal research of Chiao *et al.*,[1] Kelley,[2] Alfano and Shapiro,[3] and Gustafson *et al.*[4] The propagation of Gaussian pulses in homogeneous and lenslike media has been solved in the linear regime by Kogelnik and Li[5] and Tien *et al.*[6] We recently[7] addressed the problem of the propagation of a pulse in a medium with nonlinearity and with a parabolic graded-index profile, neglecting dispersion and self-steepening. We found that the spectral shape is qualitatively and quantitatively different from that of conventional theory; furthermore, we proved that under certain conditions the blue may lead the red in the supercontinuum, thus opening the possibility for pulse compression without external gratings. The question can then be posed whether it is necessary to have a graded-index material to observe these new effects. In this Letter we use our results from Ref. 7 to study the competing effects of self-focusing and diffraction on the spectral distribution of the supercontinuum generated by the propagation of an ultrafast pulse in a homogeneous medium. The spectral distribution is shown to differ from those predicted by the conventional self-phase-modulation theory. Furthermore, we also find that the blue can lead the red in an extended range of experimental parameters.

In the slowly varying approximation, the envelope of the electrical field can be written for $r < a$ as[8]

$$\epsilon(r, z, u) = \frac{\tilde{\epsilon}_0}{\omega(z, u)} \times \exp\left[-\frac{r^2}{a^2\omega^2(z, u)} - i\frac{k}{2}\rho(z, u)r^2 + ik\alpha(z, u)\right], \quad (1)$$

where $u = (z/v_g - t)$ is the comoving coordinate, v_g is the group velocity, a is the initial beam radius assuming a Gaussian spatial distribution, $\tilde{\epsilon}_0 = \epsilon_0 \exp(-u^2/2\tau^2)$, ϵ_0 is the electric-field initial amplitude, and τ is the pulse duration assuming an initial Gaussian temporal distribution. The physical parameters are: ω the normalized beam radius, ρ the inverse radius of curvature, and $k\alpha$ the longitudinal phase.

The solutions for ω, ρ, α, with the boundary conditions $\omega = 1$, $\rho = 0$, and $\alpha = 0$ at $z = 0$ for $r/a \ll 1$, for the case of a homogeneous medium as deduced from Ref. 7 are given by

$$\omega = [1 + y^2(1 - p')]^{1/2}, \quad (2)$$

$$L_d\rho = \frac{y(1 - p')}{[1 + y^2(1 - p')]}, \quad (3)$$

$$k\alpha = \frac{(1 - \frac{1}{2}p')}{(1 - p')^{1/2}} \arctan[(1 - p')^{1/2}y], \quad (4)$$

for $p' < 1$, and

$$k\alpha = \frac{(1 - \frac{1}{2}p')}{(p' - 1)^{1/2}} \ln\left[\frac{1 + y(p' - 1)^{1/2}}{1 - y(p' - 1)^{1/2}}\right], \quad (5)$$

for $p' > 1$, where $p' = p\exp(-u^2/\tau^2)$, $p = \epsilon_0{}^2/\epsilon_c{}^2$, $y = z/L_d$, L_d is the Rayleigh diffraction length, $1/L_d = 2/a^2k$, and $\epsilon_c{}^2 = n_0a^2/2n_2L_d{}^2 = n_0\lambda^2/2n_2\pi^2a^2$. ϵ_c is the critical field for self-focusing. In the above notation the normalized self-focusing distance is $y_{\text{foc}} = 1/(p - 1)^{1/2}$.

In what follows, the analysis is limited to $y < y_{\text{foc}}$, where the above solutions are valid. It should also be noted that for $p \gg 1$ the beam profile deviates substantially from a Gaussian profile, and the errors in the self-focal distance and in the critical power value between the numerically computed values and those predicted from the self-similar approximation can be 40 and 75%, respectively.[8] Therefore, for $p > 1$, the self-similar solutions obtained here are only qualitatively correct.

The solutions for ω, ρ, and α in the linear regime ($\epsilon_0 \to 0$) reduce to the standard formula of a Gaussian pulse propagating in a homogeneous medium,[5,6]

$$\omega = (1 + y^2)^{1/2}, \quad (6)$$

0146-9592/88/121090-03$2.00/0

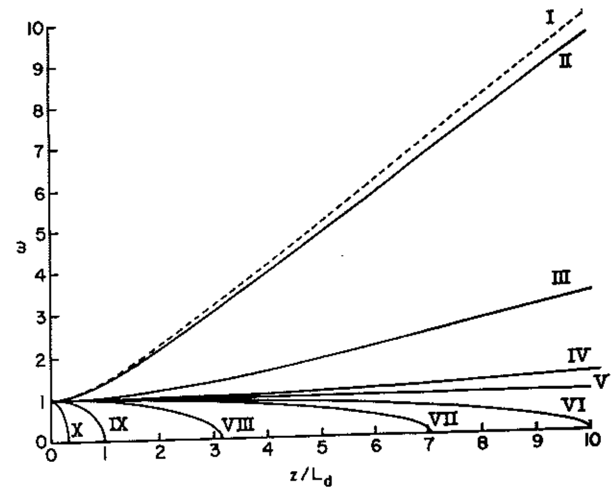


Fig. 1. Normalized beam radius for $u = 0$ as a function of the normalized length (z/L_d). I, $p \ll 1$; II, $p = 0.1$; III, $p = 0.9$; IV, $p = 0.99$; V, $p = 1$; VI, $p = 1.01$; VII, $p = 1.02$; VIII, $p = 1.1$; IX, $p = 2$; X, $p = 10$.

$$\rho L_d = y/(1 + y^2), \tag{7}$$

$$k\alpha = \arctan(y) \equiv \eta(z). \tag{8}$$

The expression for the time-dependent portion of the phase for the plane-wave approximation ($a \to \infty$) reduces to that of the conventional self-phase-modulation theory,

$$\lim_{a\to\infty} k\alpha_t = -\frac{1}{2}p'y = -\frac{kn_2}{2n_0}\tilde{\epsilon}_0^{\,2} z = \phi_{\mathrm{spm}}. \tag{9}$$

Next, the general features of the beam diameter scaling factor, the inverse radius of curvature, and the phase and spectral distributions are examined for different physical regimes. The spectral distribution is the square of the amplitude of the Fourier transform of the complex electric field.

In Fig. 1 the normalized beam radius is plotted for $u = 0$ as a function of the normalized length (z/L_d) for different electric-field intensities $p = \epsilon_0^2/\epsilon_c^2$. For weak fields ($p < 1$) diffraction is the dominant effect, while for the most intense fields ($p > 1$) the nonlinearity is dominant, there is self-focusing, and ω approaches zero as the length approaches the self-focusing distance.

In Fig. 2 the normalized inverse radius of curvature ($L_d/R = \rho L_d$) is plotted for $u = 0$ as a function of the normalized length. For $p < 1$, this quantity is positive, has a maximum for $y = 1/(1 - p)^{1/2}$, and goes to zero for large distances, i.e., diffraction is the dominant effect. For $p > 1$, this quantity is negative, decreases monotonically, and approaches $-\infty$ at the self-focusing distance.

In Fig. 3 the regularized longitudinal phase, i.e., its value for a specific p minus its value for $p = 0$, is plotted for $u = 0$ as a function of the normalized length. For $0 < p < 2$, the longitudinal phase changes sign in the total length interval, and when positive it is the opposite sign of that predicted by the conventional self-phase-modulation theory. For $1 < p < 2$, this phase has a positive asymptote for the sample length equal to the self-focusing distance. For $p > 2$ this phase is everywhere negative and admits the conventional self-phase-modulation curve as a tangent at the origin; however, it decreases much faster as the length increases until it reaches a negative asymptote at the self-focusing distance.

In Fig. 4(a) the electric-field intensity (ϵ_0^2/ω^2) is plotted for $p \approx 1$ as a function of the normalized time (u/τ) for different normalized lengths of the sample. As can be observed, the pulse compresses with increasing length. Note that for large distances the dispersion effects impose a limit on the value of this compression. In Fig. 4(b) the regularized longitudinal phase is plotted for $p < 1$ as a function of the normalized time for different normalized lengths of the sample. As can be observed, for a small length the phase has the same sign as for conventional self-phase-mod-

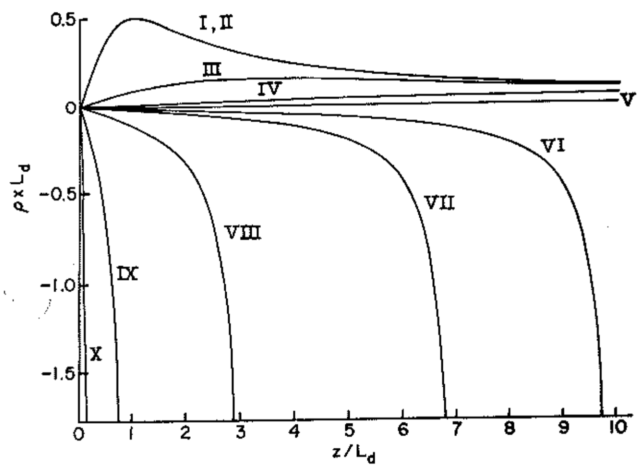


Fig. 2. Normalized inverse radius of curvature (ρL_d) for $u = 0$ as a function of the normalized length (z/L_d). I, $p \ll 1$; II, $p = 0.1$; III, $p = 0.9$; IV, $p = 0.99$; V, $p = 1$; VI, $p = 1.01$; VII, $p = 1.02$; VIII, $p = 1.1$; IX, $p = 2$; X, $p = 10$.

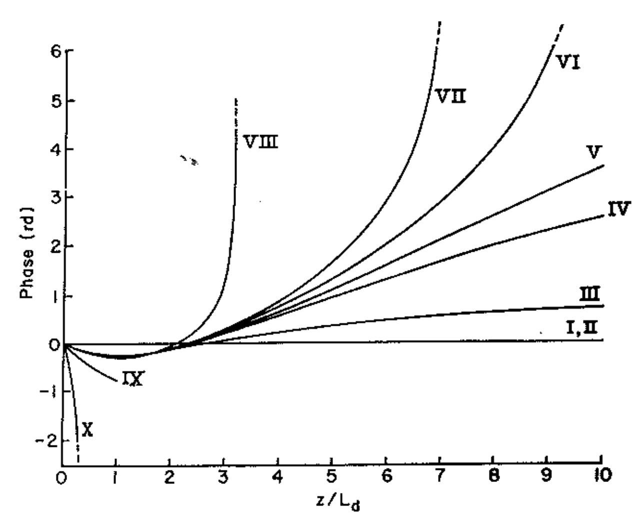
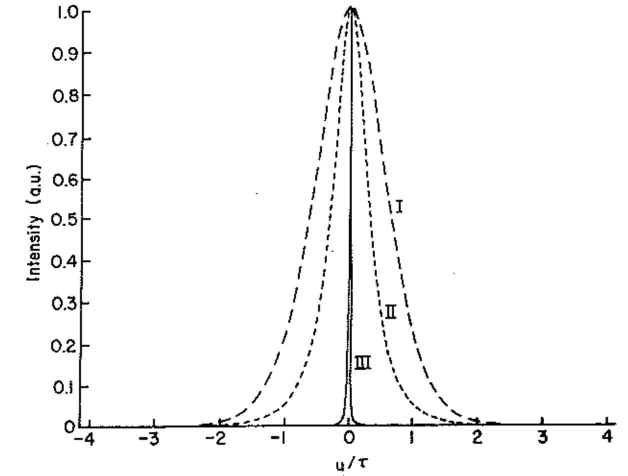
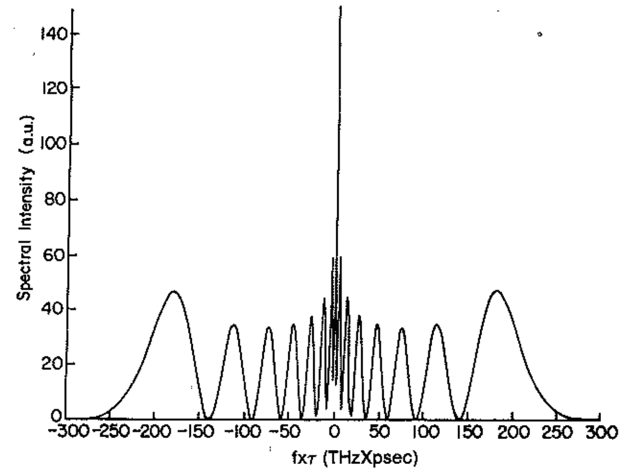


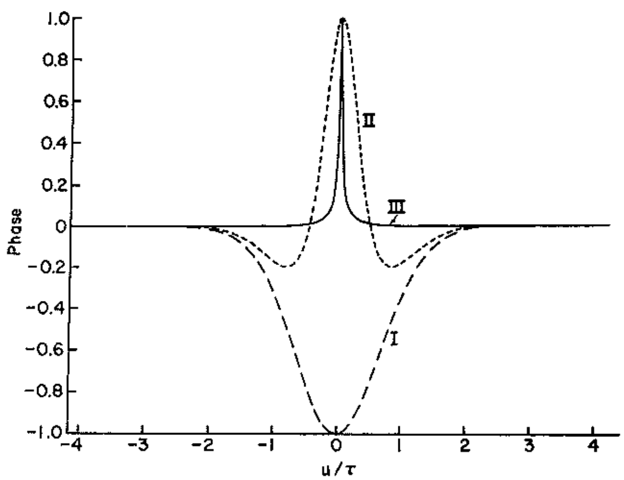
Fig. 3. Regularized longitudinal phase for $u = 0$ as a function of the normalized length (z/L_d). I, $p \ll 1$; II, $p = 0.1$; III, $p = 0.9$; IV, $p = 0.99$; V, $p = 1$; VI, $p = 1.01$; VII, $p = 1.02$; VIII, $p = 1.1$; IX, $p = 2$; X, $p = 10$.



(a)



(c)



(b)

Fig. 4. (a) Normalized intensity of the pulse as a function of time. $p = 0.99999$. I, $z/L_d = 1$; II, $z/L_d = 3$; III, $z/L_d = 100$. (b) Regularized longitudinal phase as a function of time. $p = 0.99999$. I, $Z/L_d = 1$; II, $z/L_d = 3$; III, $z/L_d = 100$. (The values of the phase peaks are -0.285, 0.25, and 46.87, respectively. The corresponding phases for self-phase-modulation theory are -0.5, -1.5, and -50, respectively.) (c) Spectral distribution intensity as a function of the normalized frequency difference. $p = 0.99999$, $z/L_d = 100$.

ulation theory, for an intermediate length the phase changes sign in its width interval, and for a large length the phase is always positive leading to the reverse of the red leading the blue in the supercontinuum. In Fig. 4(c) the spectral distribution for $p \approx 1$ and large z is plotted; note that a central peak appears in the spectrum. This effect is due to the narrowing of the α phase. This central peak was observed in an earlier paper.[3]

In conclusion, the simultaneous effects of self-focusing, self-phase modulation, and diffraction on the amplitude, phase, and spectral distribution in a $\chi^{(3)}$ medium are computed. Our theoretical results reduce to those of diffraction theory for low-intensity pulses and to conventional self-phase-modulation theory for large transverse-diameter pulses. In general, the competing effects of diffraction and self-focusing lead to novel features of the pulse shape and the generated supercontinuum.

We gratefully acknowledge the partial support of Hamamatsu Photonics K.K. and thank one of the referees for a constructive suggestion.

References

1. R. Y. Chiao, E. Garmire, and C. H. Townes, Phys. Rev. Lett. **13**, 479 (1964).
2. P. L. Kelley, Phys. Rev. Lett. **15**, 1085 (1965).
3. R. R. Alfano and S. L. Shapiro, Phys. Rev. Lett. **24**, 592 (1970).
4. T. K. Gustafson, J. P. Taran, H. A. Haus, J. R. Lifsitz, and P. L. Kelley, Phys. Rev. **177**, 306 (1969).
5. H. Kogelnik and T. Li, Proc. IEEE **54**, 1312 (1966); H. Kogelnik, Appl. Opt. **4**, 1562 (1965).
6. P. K. Tien, J. P. Gordon, and J. R. Whinnery, Proc. IEEE **53**, 129 (1965).
7. J. T. Manassah, P. L. Baldeck, and R. R. Alfano, Opt. Lett. **13**, 589 (1988).
8. J. H. Marburger, Prog. Quantum Electron. **4**, 35 (1975).