Fall 2022

Professor David Schmeltzer

TOPOLOGICAL MATERIALS-BAND THEORY

Seldom new states of matter are predicted and even less often are observed in the laboratory. This was the case with Topological Insulators like $HgTe$ observed by Konig et al. This novel material shows charge quantization and conductance quantization .

Textbook: Andrei Bernewig
Topological insulator and superconductor.

1. Introduction

1.1 The Bravais Latice
1.2 The reciprocal lattice
1.3 The Brillouine zone
1.4 Bloch electrons
1.5 A tight binding model for electrons
1.6 Discrete symmetry :Time Reversal and Inversion.

Reference for the introduction - Charles Kittel"Introduction to Solid State Physics"

Additional rerference is the article of Charles Blount " Fomalism of Band Theory" Advances of Phyics pages $305-371$

H.W.

1)Find the representation of gas of electrons in the tight binding approximation in one dimensions
2) Construct the tight binding approximation when the nearest neighbors alternate,$t+\delta t_1$ and $t-\delta t_1$
3)Find the Time Reversal state for a free particle spin $\frac{1}{2}$.

2.Topology and band theory

2.1 Topology
2.2 Band Theory
2.3 Bulk-Boundary correspondence
2.4 The Quantum Adiabatic theorem and Berry's phase
2.5 Berry Phase and Chern invariant

Reference for the Topology and band theory - Bary Simon P.R. L .volume 51 **Number** 24**page** 2167

Additional refeence is the article of Zak" Magnetic Translation Goup" P.R.B. volume $1346A$ **pages**$1602-A1606$

H.W.

1)Semiclassical approximation; adiabatic transport , compute the Berry phase.
2) Compute the Berry Phase for a two level system
3)Compute the curvature for the two level system

3. Polarization and Topology in one dimension

3.1 Polarization as a Berry Phase
3.2 Su Schrieffer Heeger model
3.3 Domain Wall States
3.4 Thouless Charge Pump

Reference J.Zak .Phys.Rev.Lett. 62 2747 (2007)

H.W.

1.Complete the detail for the Su Schrieffer Heeger model and compute the Berry phase

4. Bulk Boundary correpondence- example

4.1The conductivity for the Hall effect.
4.2 The relation between the conductivity and the Berry curvature
4.3 Haldane Model
4.4 Chiral Edge States
4.5 For a system with time reversal broken the $Kubo$ conductivity can be represented as an integral with respect the Berry's curvature.

Reference Thouless Phys.Rev.Lett. 49 405 (1982)

H.W.

1.Compute the conductivity for $h = \vec{\sigma} \cdot (-i\vec{\partial_r}) + \sigma_3 m$ in $2+1$ dimensions. Use the relation between the $Kubo$ conductivity and the integral with respect the Berry's curvature.

5.Topological Insulator for Time Reversal and Inversion invariant systems

5.1 Spin Hall Insulator
5.2 Z_2 Topological invariant
5.3 Topological Insulators in three Dimensions
5.4 The analog to the kubo formula is the non-linear response.
5.5 The integral with respect the Berry's curvature is zero ,but the second Chern number C_2 is non-zero.
5.6 The kubo non-linear response is given by the integral with respect the second Chern number C_2
5.7 The Z_2 invariant-the Chales Kane for computing the topological index using the winding of the time reversal and inversion invariant points.

Reference Fu and Kane PRB 76 045302 (20075)

H.W.

1.Use the C.Kane formula to compute the topological index for time reversal invariant topological insulator in $3+1$ dimensions .

6. Related topics (If time permits)

6.1 Crystaline Insultors
6.2 Topological Superconductors
6.3 Nodal Semimetal

Prerequisites: Quantum Mechanics

Grading policy

1)Homework will be collected each week.
2)Students will prepare a report of a paper related to the course.

Describtion of the course

Seldom new states of matter are predicted and even less often are observed in the laboratory. This was the case with Topological Insulators like $HgTe$ observed by Konig et al. .

The aim of the course the expose the students to study of physics where topology plays the major role

Contrary to traditional physics where the Hamiltonian dictates the physics in topological material boundary determine the physics

a)A central goal in Condensed Matter is to characterizes phases of matter.A magnet or a Superconductor can be understood in terms of the symmetries that they spontaneusly break. It was become clear that a concept of entaglement must be used to describe the Quantum Hall effect and topological order.

b)A central notion in the theory of electrons is the formation of bands which are characterized by the concept of $Topological\ Equivalence$:The surface of a sphere ($g = 0$) and a doughnut ($g = 1$ are distinguishable topologically. The Mobius strip and the Mobius space are obtained by identifying the left and the right edge of a sheet of paper giving it a twist

c)The Bravais Lattice is obtained once the Hamiltonian obey $H(\vec{r}) = H(\vec{r} + n_1 a_1 + n_2 a_2 + n_3 a_3)$, where n_i are integer and a_i are unit vectors.

In a periodic potential the $Bloch$ theorem guaranties that in momentum space with the momentum $\vec{K} = k_1 b_1 + k_2 b_2 + k_3 b_3$ with the $reciprocal$ unit vectors $b_3 = \vec{a}_1 \times \vec{a}_2/|a_3|$ obey $\Psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} U_{\vec{k}}(\vec{r})$

In momentum space we have the periodicity $H(\vec{k}) = H(\vec{k} + \vec{G})$ where $\vec{G}$ is the reciprocal vector.

d)The charge and conductance quantization are related to discrete singularities in the Brilouine Zone . Quantization follows from obstruction,in particular It is not possible to fix the phase for all the Billouine Zone. This leads to topological term named $Chern\ Invariants$. In particular k and $x = i\partial_k$ do not commute in this way we can intoduce the *Berry phase* In a spin dependent space we have the wave function $|U_\alpha(\vec{K} >$ which allows to introduce the spin connections.

$A_a^{\alpha,\beta}(\vec{k}) =< U_\beta(\vec{K})|\partial_a|U_\alpha(\vec{K}) >$ which is the *Berry* phase.

$A_a^{\alpha,\beta}(\vec{k})$ allows to compute the curvature and connect the result to a simple geometrical describtion

e) The point symmetry such as rotation and reflection and discrete symmetry as time reversal symmetry.The evolution is symmetric in past and future.,such concepts are important for defining Topological Invariants.

The prerequisite is. Any course which cover elements of Quantum Physics is acceptable.

Number of Credits is 4

The Prerequisite is knowledge of *Quantum Physics*.Any course which cover this material is acceptable.

The corequisite of the following courses is acceptable.
Student can take in parallel the followings:
a)551-Quantum I with the Pre:Math 391 and 346 ,Phys 351
b)552-Quantum II with the Pre:Phys 551 and ,Phys 361
c)554Solids State Physics with the Pre: Phys 551 or Phys 321