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WHY NONCONSERVATIVE INTERFACE ALGORITHMS MAY BE APPLICABLE: ANALYSIS FOR CHIMERA GRIDS¹

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Abstract

In computations of flowfields with discontinuities by zonal methods, it is believed that conservation at grid interfaces is necessary to guarantee the correct discontinuity locations and jumps. In this paper we investigate the calculations of 3D systems of conservation laws using nonconservative treatments at interfaces between Chimera embedding grids. Under some assumptions, we prove that conservation error due to nonconservative interface schemes is bounded and tends to zero as mesh size approaches zero. We also show the limit of the solutions obtained by such schemes are weak solutions of the systems. A numerical example is presented to illustrate these conclusions.

Introduction

In simulating flows over complex geometries by zonal methods, it is crucial to choose proper algorithms for grid interfaces. Grid interface algorithms fall into two categories: conservative treatments and nonconservative treatments¹.

A conservative treatment ensures conservation of mass, momentum, and energy at grid interfaces in calculations of Euler equations, and, if a numerical solution converges, it guarantees that the solution converges to a weak solution of the equations. As a result, conservation at the interfaces is now widely claimed and its procedure has been studied by many authors, cf. [2,3,4]. Nevertheless, there are some negative aspects about them, cf. [5,6]. Most important of all, implementing the conservation is very complicated, if not impossible, in 2- and 3-dimensional Chimera embedding grids, which are often encountered in calculations with complex geometries.

There have been systematic studies about nonconservative treatments for grid interfaces as well as many papers reporting their successful applications, cf. [7,8]. However, it is believed that a conservative treatment between grids is necessary to ensure correct discontinuity locations and jumps, and numerical examples do show that a nonconservative treatment may cause inaccurate shock jumps and shock locations, especially when zonal boundaries are close to the exact shock locations (see [9]). As indicated in [4], it is unclear about what the effects of a nonconservative interface condition would be on the overall accuracy and correctness of numerical solutions.

Therefore, it is natural to ask why nonconservative treatments may be used and whether they can provide right jumps and locations of discontinuities. The authors have investigated nonconservative

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conditions at grid interfaces and given explanations to these questions¹⁰. In the current paper, we present the explanations via two theorems and illustrate them with a numerical example.

Governing Equations and Their Discretization

Consider the calculation of the initial value problem of a 3D system of conservation laws

$$\begin{aligned} U_t(x, y, z, t) + F_x(U(x, y, z, t)) + G_y(U(x, y, z, t)) + H_z(U(x, y, z, t)) &= 0, \\ U(x, y, z, 0) &= U_0(x, y, z). \end{aligned} \quad (1)$$

Here, $t > 0, x, y, z \in \Omega$, $\Omega = \langle x, y, z \mid x, y, z = -\infty, +\infty \rangle$, $U_0(x, y, z)$ has a compact support, and, for convenience, it is assumed that $F(0) = G(0) = H(0) = 0$. Let Ω be divided into Ω_A and Ω_B by a zonal interface Σ (Fig. 1). In Ω_A the system is written in coordinate (x, y, z, t) , while in Ω_B it is written in coordinate $(\xi, \eta, \varsigma, t)$,

$$\bar{U}_t(\xi, \eta, \varsigma, t) + \bar{F}_\xi(\bar{U}(\xi, \eta, \varsigma, t)) + \bar{G}_\eta(\bar{U}(\xi, \eta, \varsigma, t)) + \bar{H}_\varsigma(\bar{U}(\xi, \eta, \varsigma, t)) = 0. \quad (2)$$

On Σ $\xi = \text{const}$.

As depicted in Fig. 1, the calculation on Ω_A and Ω_B is done on grid G_A and G_B , respectively, which are comprised of the lines $x, y, z = \text{const}$ and lines $\xi, \eta, \varsigma = \text{const}$, respectively. The two grids overlap in an arbitrary way, and they are so fine that Σ is nearly a plane within each cell of G_A . Let $\{i_B, j_B, k_B\}$ be nodes of G_B involved in the calculation on Ω_B , and $\{i_{BI}, j_{BI}, k_{BI}\}$ be its nodes located on Σ . In order to obtain the solutions at the nodes of G_A that face and are next to Σ (represented by circles in Fig. 1), conditions are needed at some of its nodes in Ω_B (marked with dots in Fig. 1). For each j, k there is a such node $(i_A(j, k), j, k)$, for each i, k a node $(i, j_A(i, k), k)$, and for each i, j a node $(i, j, k_A(i, j))$. These nodes make up $\{i_{AI}, j_{AI}, k_{AI}\}$. Let all the nodes of G_A involved in the calculation on Ω_A be $\{i_A, j_A, k_A\}$. Furthermore, let Σ_A be a surface comprised by many small surfaces

$$\begin{aligned} \langle x, y, z \mid x = x_{i_A(j, k)-1/2, j, k}, y, z \in (y_{i_A(j, k), j-1/2, k}, y_{i_A(j, k), j+1/2, k}] \times (z_{i_A(j, k), j, k-1/2}, z_{i_A(j, k), j, k+1/2}) \rangle, \\ \langle x, y, z \mid y = y_{i, j_A(i, k)-1/2, k}, x, z \in (x_{i-1/2, j_A(i, k), k}, x_{i+1/2, j_A(i, k), k}] \times (z_{i, j_A(i, k), k-1/2}, z_{i, j_A(i, k), k+1/2}) \rangle, \\ \langle x, y, z \mid z = z_{i, j, k_A(i, j)-1/2}, x, y \in (x_{i-1/2, j, k_A(i, j)}, x_{i+1/2, j, k_A(i, j)}) \times (y_{i, j-1/2, k_A(i, j)}, y_{i, j+1/2, k_A(i, j)}) \rangle, \end{aligned} \quad (3)$$

and Σ_B be the surface $\xi = 1/2$. It is readily seen that when mesh size goes to zero, Σ_A will overlap with Σ .

We define that $\Omega_{i, j, k}^A = \langle x, y, z \mid x, y, z \in (x_{i-1/2}, x_{i+1/2}] \times (y_{j-1/2}, y_{j+1/2}] \times (z_{k-1/2}, z_{k+1/2}) \rangle$, $i, j, k \in \{i_A, j_A, k_A\}$, $\Omega_{i, j, k}^B = \langle \xi, \eta, \varsigma \mid \xi, \eta, \varsigma \in (\xi_{i-1/2}, \xi_{i+1/2}] \times (\eta_{j-1/2}, \eta_{j+1/2}] \times (\varsigma_{k-1/2}, \varsigma_{k+1/2}) \rangle$, $i, j, k \in \{i_B, j_B, k_B\}$, $\Omega_{AD} = \bigcup \Omega_{i, j, k}^A$, $i, j, k \in \{i_A, j_A, k_A\} \setminus \{i_{AI}, j_{AI}, k_{AI}\}$, $\Omega_{BD} = \bigcup \Omega_{i, j, k}^B$, $i, j, k \in \{i_B, j_B, k_B\} \setminus \{i_{BI}, j_{BI}, k_{BI}\}$, $\Omega_I = \Omega \setminus \Omega_{AD} \setminus \Omega_{BD}$. Then, based on some volume-wise constant functions, on G_A and G_B the formulation of explicit conservative schemes with a 3 point stencil can be written as, respectively,

$$\begin{aligned} U_\Delta^A(x, y, z, t + \Delta t) &= U_\Delta^A(x, y, z, t) - \frac{\Delta t}{\Delta x} (F_\Delta(x + \Delta x/2, y, z, t) - F_\Delta(x - \Delta x/2, y, z, t)) \\ &\quad - \frac{\Delta t}{\Delta y} (G_\Delta(x, y + \Delta y/2, z, t) - G_\Delta(x, y - \Delta y/2, z, t)) \\ &\quad - \frac{\Delta t}{\Delta z} (H_\Delta(x, y, z + \Delta z/2, t) - H_\Delta(x, y, z - \Delta z/2, t)), \end{aligned} \quad (4)$$

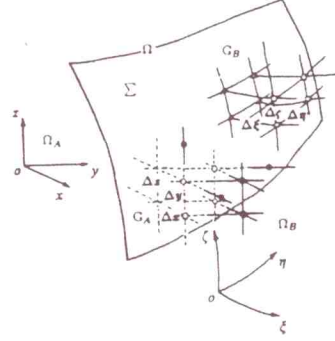


Fig. 1 Ω_A and Ω_B and their grids

and

$$U_{\Delta}^B(\xi, \eta, \varsigma, t + \Delta t) = U_{\Delta}^B(\xi, \eta, \varsigma, t) - \frac{\Delta t}{\Delta \xi} (\bar{F}_{\Delta}(\xi + \Delta \xi/2, \eta, \varsigma, t) - \bar{F}_{\Delta}(\xi - \Delta \xi/2, \eta, \varsigma, t)) \\ - \frac{\Delta t}{\Delta \eta} (\bar{G}_{\Delta}(\xi, \eta + \Delta \eta/2, \varsigma, t) - \bar{G}_{\Delta}(\xi, \eta - \Delta \eta/2, \varsigma, t)) \\ - \frac{\Delta t}{\Delta \varsigma} (\bar{H}_{\Delta}(\xi, \eta, \varsigma + \Delta \varsigma/2, t) - \bar{H}_{\Delta}(\xi, \eta, \varsigma - \Delta \varsigma/2, t)). \quad (5)$$

At $\{i_{AI}, j_{AI}, k_{AI}\}$ and $\{i_{BI}, j_{BI}, k_{BI}\}$, we use a certain explicit spatial interpolation that is not necessarily conservative.

Lemma 1 Suppose

$$\|U_{\Delta}^A(x, y, z, t)\| < m_1, \quad \|U_{\Delta}^B(\xi, \eta, \varsigma, t)\| < m_2, \quad (6)$$

then

$$\|F_{\Delta}(x, y, z, t)\| < m_3, \quad \|\bar{F}_{\Delta}(\xi, \eta, \varsigma, t)\| < m_6, \\ \|G_{\Delta}(x, y, z, t)\| < m_4, \quad \|\bar{G}_{\Delta}(\xi, \eta, \varsigma, t)\| < m_7, \\ \|H_{\Delta}(x, y, z, t)\| < m_5, \quad \|\bar{H}_{\Delta}(\xi, \eta, \varsigma, t)\| < m_8, \quad (7)$$

where $m_1 - m_8$ are constants. Moreover, suppose $U_{\Delta}(x, y, z, t)$ converges uniformly to some function $U(x, y, z, t)$ as $\Delta \rightarrow 0$, where $\Delta = \max(\Delta x, \Delta y, \Delta z, \Delta \xi, \Delta \eta, \Delta \varsigma, \Delta t)$. Then, at the place where $U(x, y, z, t)$ is continuous,

$$\lim_{\Delta \rightarrow 0} F_{\Delta}(x, y, z, t) = F(U(x, y, z, t)), \\ \lim_{\Delta \rightarrow 0} G_{\Delta}(x, y, z, t) = G(U(x, y, z, t)), \\ \lim_{\Delta \rightarrow 0} H_{\Delta}(x, y, z, t) = H(U(x, y, z, t)), \\ \lim_{\Delta \rightarrow 0} \bar{F}_{\Delta}(\xi, \eta, \varsigma, t) = \bar{F}(\bar{U}(\xi, \eta, \varsigma, t)), \\ \lim_{\Delta \rightarrow 0} \bar{G}_{\Delta}(\xi, \eta, \varsigma, t) = \bar{G}(\bar{U}(\xi, \eta, \varsigma, t)), \\ \lim_{\Delta \rightarrow 0} \bar{H}_{\Delta}(\xi, \eta, \varsigma, t) = \bar{H}(\bar{U}(\xi, \eta, \varsigma, t)), \quad (8)$$

and at the place it is discontinuous

$$\|\lim_{\Delta \rightarrow 0} F_{\Delta}(x, y, z, t) - F(U(x, y, z, t))\| \leq \|O(U(x^+, y, z, t) - U(x^-, y, z, t))\|, \\ \|\lim_{\Delta \rightarrow 0} G_{\Delta}(x, y, z, t) - G(U(x, y, z, t))\| \leq \|O(U(x, y^+, z, t) - U(x, y^-, z, t))\|, \\ \|\lim_{\Delta \rightarrow 0} H_{\Delta}(x, y, z, t) - H(U(x, y, z, t))\| \leq \|O(U(x, y, z^+, t) - U(x, y, z^-, t))\|, \\ \|\lim_{\Delta \rightarrow 0} \bar{F}_{\Delta}(\xi, \eta, \varsigma, t) - \bar{F}(\bar{U}(\xi, \eta, \varsigma, t))\| \leq \|O(\bar{U}(\xi^+, \eta, \varsigma, t) - \bar{U}(\xi^-, \eta, \varsigma, t))\|, \\ \|\lim_{\Delta \rightarrow 0} \bar{G}_{\Delta}(\xi, \eta, \varsigma, t) - \bar{G}(\bar{U}(\xi, \eta, \varsigma, t))\| \leq \|O(\bar{U}(\xi, \eta^+, \varsigma, t) - \bar{U}(\xi, \eta^-, \varsigma, t))\|, \\ \|\lim_{\Delta \rightarrow 0} \bar{H}_{\Delta}(\xi, \eta, \varsigma, t) - \bar{H}(\bar{U}(\xi, \eta, \varsigma, t))\| \leq \|O(\bar{U}(\xi, \eta, \varsigma^+, t) - \bar{U}(\xi, \eta, \varsigma^-, t))\|. \quad (9)$$

Proof Using mean-value theorem and noticing consistency and construction of numerical fluxes, the proof is obvious.

Conservation Error

Since $U_0(x, y, z)$ has a compact support, and, in view that the schemes both for inner nodes and interface nodes are explicit with finite stencils, for a fixed time T there must be a bounded region Ω_T outside which $U_{\Delta}(x, y, z, t)$ ($0 < t \leq T$) always equals zero. For convenience, let $\Sigma_{Tz} = \langle y, z \mid y, z \in \Sigma \cap \Omega_T \rangle$, $\Sigma_{Ty} = \langle x, z \mid x, z \in \Sigma \cap \Omega_T \rangle$, $\Sigma_{Tx} = \langle x, y \mid x, y \in \Sigma \cap \Omega_T \rangle$, $\Sigma_{T\xi} = \langle \eta, \varsigma \mid \eta, \varsigma \in \Sigma \cap \Omega_T \rangle$. The numerical approximation of $U(x, y, z, t)$ in (1) over Ω can be defined as

$$U_{\Delta}(x, y, z, t) = \begin{cases} U_{\Delta}^A(x, y, z, t), & x, y, z \in \Omega_A, \\ U_{\Delta}^B(\xi, \eta, \varsigma, t), & \xi, \eta, \varsigma \in \Omega_B. \end{cases} \quad (10)$$

Integrating (1) shows that the solution of (1) has a so-called conservation property:

$$\int_{\Omega} (U(x, y, z, t_2) - U(x, y, z, t_1)) dx dy dz = 0 \quad (11)$$

for any $t_1, t_2 > 0$. However, usually this is not true for $U_\Delta(x, y, z, t)$ and we define its conservation error as

$$S_\Delta(t_1, t_2) \equiv \int_\Omega (U_\Delta(x, y, z, t_2) - U_\Delta(x, y, z, t_1)) dx dy dz. \quad (12)$$

If algorithms at $\{i_{AI}, j_{AI}, k_{AI}\}$ and $\{i_{BI}, j_{BI}, k_{BI}\}$ are designed so that $S_\Delta(t_1, t_2) = 0$, they are the so-called conservative schemes, cf. [2, 3, 4]. Otherwise, they are nonconservative interface algorithms. As shown in [10], conservation error is a kind of difference between exact and computed solutions and may lead inaccuracy of discontinuity jumps and locations. For a computation of the problem (1) on $\Omega \times (0, T]$, conservation error is $S_\Delta(0, T)$. By straightforward calculations, it is obtained that

$$\begin{aligned} S_\Delta(0, T) &= \int_{\Omega_{AD} \cap \Omega_T} (U_\Delta(x, y, z, T) - U_\Delta(x, y, z, 0)) dx dy dz \\ &\quad + \int_{\Omega_{BD} \cap \Omega_T} (U_\Delta(\xi, \eta, \zeta, T) - U_\Delta(\xi, \eta, \zeta, 0)) d\xi d\eta d\zeta \\ &\quad + \int_{\Omega_I \cap \Omega_T} (U_\Delta(x, y, z, T) - U_\Delta(x, y, z, 0)) dx dy dz \\ &= D(T) + I(T), \end{aligned} \quad (13)$$

where

$$\begin{aligned} D(T) &= \int_{\Sigma_{Tx} \times (0, T]} F_\Delta(x^{\Sigma_A}, y, z, t) dy dz dt + \int_{\Sigma_{Ty} \times (0, T]} G_\Delta(x, y^{\Sigma_A}, z, t) dx dz dt \\ &\quad + \int_{\Sigma_{Tz} \times (0, T]} H_\Delta(x, y, z^{\Sigma_A}, t) dx dy dt - \int_{\Sigma_{T\xi} \times (0, T]} \bar{F}_\Delta(\xi^{\Sigma_B}, \eta, \zeta, t) d\eta d\zeta dt, \\ I(T) &= \int_{\Omega_I \cap \Omega_T} (U_\Delta(x, y, z, T) - U_\Delta(x, y, z, 0)) dx dy dz \end{aligned} \quad (14)$$

Theorem 1 Let $U_\Delta(x, y, z, t)$ be uniformly bounded. Then, 1) $S_\Delta(0, T)$ is bounded; 2) $S_\Delta(0, T)$ tends to zero when $U_\Delta(x, y, z, t)$ converges to some function $U(x, y, z, t)$ and either of the following two conditions is satisfied,

A $U_\Delta(x, y, z, t)$ converges uniformly near Σ and its limit is continuous within $\Sigma_{T\xi} \times (0, T]$, except on m surfaces, where m is at most countable;

B $U(x, y, z, t)$ is discontinuous within $\Sigma_{T\xi} \times (0, T]$, but

$$\begin{aligned} \lim_{\Delta \rightarrow 0} F_\Delta(x^{\Sigma_A}, y, z, t) &= F(U(x^{\Sigma^-}, y, z, t)), \\ \lim_{\Delta \rightarrow 0} G_\Delta(x, y^{\Sigma_A}, z, t) &= G(U(x, y^{\Sigma^-}, z, t)), \\ \lim_{\Delta \rightarrow 0} H_\Delta(x, y, z^{\Sigma_A}, t) &= H(U(x, y, z^{\Sigma^-}, t)), \\ \lim_{\Delta \rightarrow 0} \bar{F}_\Delta(\xi^{\Sigma_B}, \eta, \zeta, t) &= \bar{F}(U(\xi^{\Sigma^+}, \eta, \zeta, t)), \end{aligned} \quad (15)$$

and R-H condition holds across Σ .

Proof 1) (13) gives rise to

$$\|S_\Delta(0, T)\|_\infty \leq \|D(T)\|_\infty + \|I(T)\|_\infty. \quad (16)$$

Due to the assumption that $U_\Delta(x, y, z, t)$ is uniformly bounded, it is known by Lemma 1 that (7) holds. Let S_{Tx} , S_{Ty} , S_{Tz} , and $S_{T\xi}$ be the areas of Σ_{Tx} , Σ_{Ty} , Σ_{Tz} , and $\Sigma_{T\xi}$, respectively, it is obtained from (14) that

$$\begin{aligned} \|D(T)\|_\infty &< T(m_3 S_{Tx} + m_4 S_{Ty} + m_5 S_{Tz} + m_6 S_{T\xi}), \\ \|I(T)\|_\infty &< 2m_1 S_{T\xi} \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} + m_2 S_{T\xi} \Delta \xi, \end{aligned} \quad (17)$$

where $m_1 - m_6$ are constants given in (6) and (7). As a result of (17), $S_\Delta(0, T)$ is bounded.

2) Since (6) and (7) hold, Lebesgue bounded convergence theorem that allows the change of the orders of limiting and integrating gives rise to

$$\begin{aligned} \lim_{\Delta \rightarrow 0} D(T) &= \int_{\Sigma_{Tx} \times (0, T]} \lim_{\Delta \rightarrow 0} F_\Delta(x^{\Sigma_A}, y, z, t) dy dz dt \\ &\quad + \int_{\Sigma_{Ty} \times (0, T]} \lim_{\Delta \rightarrow 0} G_\Delta(x, y^{\Sigma_A}, z, t) dx dz dt \\ &\quad + \int_{\Sigma_{Tz} \times (0, T]} \lim_{\Delta \rightarrow 0} H_\Delta(x, y, z^{\Sigma_A}, t) dx dy dt \\ &\quad - \int_{\Sigma_{T\xi} \times (0, T]} \lim_{\Delta \rightarrow 0} \bar{F}_\Delta(\xi^{\Sigma_B}, \eta, \zeta, t) d\eta d\zeta dt. \end{aligned} \quad (18)$$

In Fig. 1, it is readily seen that

$$\bar{F}_\Delta(\xi^{\Sigma_B}, \eta, \zeta, t) = \bar{F}_\Delta(\xi^\Sigma, \eta, \zeta, t). \quad (19)$$

In the same figure, on some parts of Σ_{T_x} , Σ_{T_y} , and Σ_{T_z} one has, respectively,

$$\begin{aligned} F_{\Delta}(x^{\Sigma_A}, y, z, t) &= F_{\Delta}(x^{\Sigma}, y, z, t), \\ G_{\Delta}(x, y^{\Sigma_A}, z, t) &= G_{\Delta}(x, y^{\Sigma}, z, t), \\ H_{\Delta}(x, y, z^{\Sigma_A}, t) &= H_{\Delta}(x, y, z^{\Sigma}, t), \end{aligned} \quad (20)$$

and on the other parts one has, respectively,

$$\begin{aligned} F_{\Delta}(x^{\Sigma_A}, y, z, t) &= F_{\Delta}(x^{\Sigma}, y, z, t) + O(\max U(x^{\Sigma_A} + l\Delta x, y, z, t) - \min U(x^{\Sigma} + l\Delta x, y, z, t)), \\ G_{\Delta}(x, y^{\Sigma_A}, z, t) &= G_{\Delta}(x, y^{\Sigma}, z, t) + O(\max U(x, y^{\Sigma_A} + l\Delta y, z, t) - \min U(x, y^{\Sigma} + l\Delta y, z, t)), \\ H_{\Delta}(x, y, z^{\Sigma_A}, t) &= H_{\Delta}(x, y, z^{\Sigma}, t) + O(\max U(x, y, z^{\Sigma_A} + l\Delta z, t) - \min U(x, y, z^{\Sigma} + l\Delta z, t)). \end{aligned} \quad (21)$$

Here $l = -1/2, 1/2$. If **A** is satisfied, it is derived from Lemma 1, (20), and (21) that

$$\begin{aligned} \lim_{\Delta \rightarrow 0} F_{\Delta}(x^{\Sigma_A}, y, z, t) &= F(U(x^{\Sigma}, y, z, t)), \\ \lim_{\Delta \rightarrow 0} G_{\Delta}(x, y^{\Sigma_A}, z, t) &= G(U(x, y^{\Sigma}, z, t)), \\ \lim_{\Delta \rightarrow 0} H_{\Delta}(x, y, z^{\Sigma_A}, t) &= H(U(x, y, z^{\Sigma}, t)), \\ \lim_{\Delta \rightarrow 0} \bar{F}_{\Delta}(\xi^{\Sigma_B}, \eta, \zeta, t) &= \bar{F}(U(\xi^{\Sigma}, \eta, \zeta, t)), \end{aligned} \quad (22)$$

except on some areas with zero measure. Using (22) and relation between coordinate system (x, y, z) and (ξ, η, ζ) yields

$$\lim_{\Delta \rightarrow 0} D(T) = 0. \quad (23)$$

If **B** is satisfied, one can verify that (23) also holds. Using (13), (23), and (17) one then has

$$\lim_{\Delta \rightarrow 0} S_{\Delta}(0, T) = 0 + 0 = 0. \quad (24)$$

This completes the proof of the theorem.

Remark 1 Condition of **A** corresponds to situations in which discontinuities move across zonal boundaries, while condition of **B** relates to cases when discontinuities stop at zonal boundaries.

Remark 2 If numerical solution reach a steady state at a neighborhood of Σ when $t \geq T$, it is more proper to define conservation error as the difference between numerical fluxes on Σ_A and those on Σ_B , that is

$$\begin{aligned} S'_{\Delta}(T) &= \int_{\Sigma_{T_x}} F_{\Delta}(x^{\Sigma_A}, y, z, T) dydz + \int_{\Sigma_{T_y}} G_{\Delta}(x, y^{\Sigma_A}, z, T) dx dz \\ &\quad + \int_{\Sigma_{T_z}} H_{\Delta}(x, y, z^{\Sigma_A}, T) dx dy - \int_{\Sigma_{T_t}} \bar{F}_{\Delta}(\xi^{\Sigma_B}, \eta, \zeta, T) d\eta d\zeta. \end{aligned} \quad (25)$$

Let $T_1, T_2 \geq T$, one may verify that $S'_{\Delta}(T) \equiv S_{\Delta}(T_1, T_2)/(T_2 - T_1)$, which shows that Theorem 1 can also be applied to a steady case.

Weak Solution

Following Lax and Wendroff [11] and the proof of Theorem 1, we can prove (for details, refer to [10])

Theorem 2 Let

$$\begin{aligned} U_{\Delta}^A(x, y, z, t) &= \frac{1}{\Delta x \Delta y \Delta z} \int_{\Omega_{i,j,k}^A} U_0(x, y, z) dx dy dz, \quad x, y, z \in \Omega_{i,j,k}^A, \quad t \in (0, \Delta t], \\ U_{\Delta}^B(\xi, \eta, \zeta, t) &= \frac{1}{\Delta \xi \Delta \eta \Delta \zeta} \int_{\Omega_{i,j,k}^B} U_0(x, y, z) dx dy dz, \quad \xi, \eta, \zeta \in \Omega_{i,j,k}^B, \quad t \in (0, \Delta t], \end{aligned} \quad (26)$$

and $U_{\Delta}(x, y, z, t)$ is uniformly bounded and converges to some function $U(x, y, z, t)$ as Δ approaches zero. Moreover, let either condition of **A** or **B** be satisfied. Then, $U(x, y, z, t)$ is a weak solution of the initial value problem (1).

Numerical Example

To illustrate above results, we show the calculation of a 3D inviscid flow field developed by a plane shock moving through a cube Ω_A with an underlying cube Ω_B (Fig. 2a). The position of Ω_A

is arranged in this way: firstly, let its center overlap with that of Ω_B and ξ, η , and ζ be parallel to x, y , and z , respectively; secondly, turn it with respect to its center an angle of α and then another angle of β . The grids for the two cubes overlap in an arbitrary way. A scheme in form of (4) and (5) and based on a TVNI scheme [12] is adopted on the two grids, and conditions at grid interfaces are given by a spatial interpolation. In Fig. 2b, 2c, compared with the exact shock that moves to the right and whose location can be found on the outer boundary of Ω_B , it is seen that, although accompanied with an amount of noise, the computed shock passes the grid interfaces smoothly and neither condition of **A** nor **B** is violated. Consequently, the numerical approximation gets closer to the exact solution and conservation error decreases as mesh size is reduced. However, numerical solutions would be wrong if both condition of **A** and **B** are not satisfied. Such an example is given in [10]. Condition **A** and **B** can be employed to make judgment about the correctness of numerical solutions obtained by nonconservative algorithms at grid interfaces.

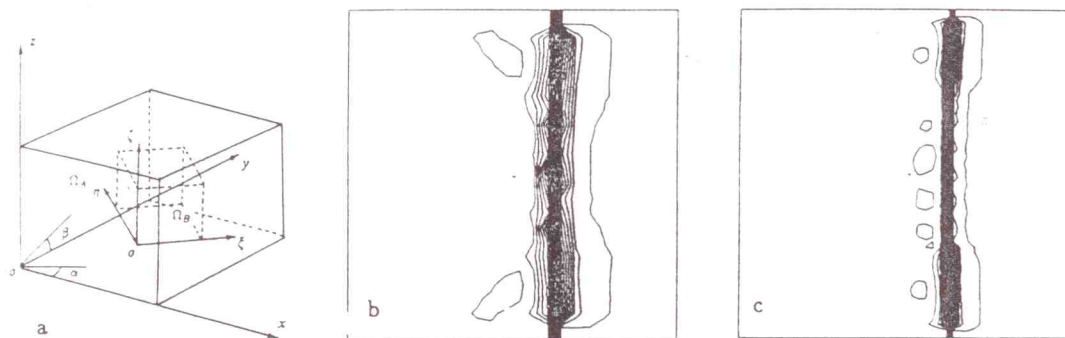


Fig. 2 Calculation of a shock. The shock moves in x -direction from $x = 0$. Behind the shock density, velocity, and pressure are 3.948, 4.359, and 5.005, respectively, in front of the shock they are 1.4, 3, and 1, respectively. a) Ω_A and Ω_B have a size of $0.5 \times 0.5 \times 0.5$ and $1 \times 1 \times 1$, respectively, $\alpha = -\pi/3, \beta = \pi/6$. b) Computed pressure contours on a y - z plane at $T=0.1$. Grid of Ω_A and Ω_B is $18 \times 18 \times 18$ and $24 \times 24 \times 24$, respectively, $\|S_\Delta(0, T)\|_\infty = 1.5 \times 10^{-3}$. c) Computed pressure contours on the same plane and at the same time. Grid of Ω_A and Ω_B is $36 \times 36 \times 36$ and $48 \times 48 \times 48$, respectively, $\|S_\Delta(0, T)\|_\infty = 1.3 \times 10^{-4}$.

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