Developmental Psychology
1986, Vol. 22, No. 2, 204–212

0012-1649/86/$00.75

The Genevan and Cattell-Horn Conceptions of Intelligence Compared: Early Implementation of Numerical Solution Aids

Irvin Sam Schonfeld
College of Physicians and Surgeons and The Psychiatric Epidemiology Training Program, Columbia University

The Genevan and Cattell-Horn theories of intelligence are compared. The theories are found to be similar in the following respects: Intelligence (operative intelligence and fluid ability) is conceptualized as adaptational in function; the products of everyday learning and crystallized skills reflect the impress of experience; one category of intelligence (operative intelligence, fluid ability) is conceptualized as prior or more fundamental than the other (learned products, crystallized skills). Important differences were also found: Whereas fluid ability is characterized as formless and fixed, operative intelligence is viewed as highly structured and evolving; a compensatory relation between noegenetic crystallized skills and fluid ability is hypothesized where such a relation is not conceived to exist between operative intelligence and learning. The relation of Piagetian operative level to the child's capacity to use crystallized solution procedures (aids) in making elementary numerical comparisons was investigated. Performance on quantitative comparison tasks reflecting the child's understanding of correspondence relations was highly related to operative level. It was also found that the child's capacity to implement solution aids in making quantitative comparisons was, to some extent, moderated by his or her level of operative development.

Matarazzo (1972), without explicitly describing how they are akin, suggested that there are parallels in the theories of intelligence developed by Piaget and by Cattell and Horn. It is the purpose of this article to outline the similarities in the two theories of intelligence and to examine a key difference. A brief description of the two viewpoints follows.

Cattell-Horn theory is conceptualized in terms of two major interrelated components of intelligence. In factor analytic studies, the components, fluid and crystallized abilities, manifest themselves as two "highly cooperative" second-order factors (Cattell, 1963). The crystallized factor "loads more highly those cognitive performances in which skilled judgment habits have become crystallized (whence its name) as the result of [the] earlier learning application of some prior, more fundamental general ability" (Cattell, 1963, pp. 2-3). Cattell (1963) asserted that crystallized ability also derives from the application of earlier acquired crystallized skills. In other words, the crystallized factor is conceived as "a precipitate out of experience" (Horn, 1967). Crystallized ability constitutes more than mere rote learning. It encompasses knowledge-producing, or, in Cattell's words, "noegenetic" processes that are "nurtured by acculturation" (Horn & Cattell, 1966). Thus, the crystallized factor, or Gc, loads on performance on tests of verbal comprehension, general reasoning, and semantic relations.

The fluid factor, or Gf, is conceptualized as the "more fundamental," biologically rooted adaptational ability. Horn (1967) characterized it as "formless" and capable of flowing into "a wide variety of intellectual activities." Fluid ability manifests itself in performance on "tests requiring adaptation to new situations, where crystallized skills are of no particular advantage" (Cattell, 1963, p. 3). It is "relatively independent of education" (Horn, 1967). Thus, fluid ability is thought to be reflected in performance on culture-fair tests like Raven's Progressive Matrices (Cattell, 1965) or on other nonverbal tests of figural reasoning like the various versions of the block design found in the Wechsler scales (Horn, 1976, 1979). Heredity is thought to make a greater contribution to fluid than to crystallized ability (Cattell, 1963, 1965; Horn, 1978; Horn & Cattell, 1966; Jensen, 1969, 1973).

In Genevan theory there are approximate analogues to fluid and crystallized abilities: operative intelligence and learning. Operative intelligence refers to adaptive, increasingly integrated and generalized sets of (first overt, then covert) actions (Flavell, 1963; Furth, 1969). Hooper et al. (1971) pointed out that Piagetian operative intelligence, which is often expressed in performances on the standard tests of conservation and logic, involves temporal integration and the eduction of relations, "general characteristics of fluid intelligence." Piaget, however, characterized operative intelligence, in contrast to the fixed and formless fluid ability, as highly structured and evolving.

Some interpreters of Piaget (Beilin, 1971; Flavell, 1971) have gone so far as to suggest that Genevan operative intelligence has a preformationist or maturationist cast (although the view is rejected in Geneva [Piaget, 1963]). In this vein, level of fluid ability

This article is based on a substantive reanalysis of data collected in one of the studies conducted for my doctoral dissertation. The dissertation was submitted to the City University of New York in partial fulfillment of the requirements for the PhD degree. I extend appreciation to the following individuals who read various versions of the article: Harry Beilin, Geoffrey Saxe, George Schonfeld, and Pearl Knopf. I extend special thanks to Jan Powell and an anonymous reviewer for their valuable criticisms. The reanalysis was supported by NIMH grants 5 T32 MH13043-13 and MH30906.

Correspondence concerning this article should be sent to Irvin Schonfeld, Department of Social and Psychological Foundations, School of Education, The City College of CUNY, 138th St. and Convent Avenue, New York, New York 10031.

is explicitly attributed to heredity. Moreover, fluid ability generally attains its peak between the ages of 12 and 15 (Cattell, 1971), approximately covering the period during which the child attains formal operations, the highest level of operative development within the Genevan framework.

Learning, like crystallized ability, refers to knowledge that is "a function of environmental data" (Furth, 1969, p. 269). Genevan (Inhelder, Sinclair, & Bovet, 1974) and neo-Genevan (Gholson & Beilin, 1979) theories hold that knowledge that is a function of environmental data, that is to say, everyday learning, is regulated by operative level. Similarly, in the Cattell-Horn view, contemporary crystallized ability is ultimately dependent on "formative fluid ability," fluid ability that was mobilized earlier in the individual's development (Cattell, 1963). The Cattell-Horn view, however, differs from the Genevan view on the degree of autonomy between, on one hand, knowledge and skills that result from environmental learning and, on the other, prior forms of intelligence. Within the Genevan framework learning is stage dependent (cf. Gholson & Beilin, 1979). By contrast, the concept of fluid and crystallized abilities "introduces a notion of alternative mechanisms in the performances that are commonly assumed to represent the operation of intelligence. That is, many intellectual tasks allow one to employ either fluid intelligence or crystallized intelligence to arrive at a correct answer" (Horn & Cattell, 1966, p. 255). Cattell-Horn theory provides for a compensatory relation between fluid and crystallized abilities (Horn, 1967, 1968, 1978, 1979; Horn & Cattell, 1966; Jensen, 1969). Where the application of fluid ability cannot lead to a solution of an intellectual problem, the application of crystallized skills may. Moreover, the Cattell-Horn view asserts that crystallized ability is truly knowledge producing in character and with age becomes increasingly independent of fluid ability (Horn, 1978). In yoking learning to operative level, the Genevan view gives acculturational learning relatively less autonomous character than is found in Cattell-Horn theory.

In the study to be described predictions based on the Genevan and Cattell-Horn models are contrasted. The domain of the study is cognition about quantity. Children's quantitative thought is an especially satisfactory area in which to contrast the two theories because of its location within both theories. A major theme of Piaget's writings is the progressive arithmetization of thought. Furthermore, Piaget (1965) advanced the view that conservation, possibly his most studied concept, is a fundamental constituent of quantitative thought. Cattell (1963) found that among children aged 5 to 7, fluid and crystallized factors loaded number facility almost equally, suggesting that within this period alternative fluid and crystallized mechanisms for solving numerical problems emerge. Horn (1967; Horn & Cattell, 1966) advanced the view that mathematical knowledge may be extended through the application of the individual's untutored fluid ability or the products of acculturation. These products, which include algebra and counting, can be aids to mathematical thought.

Counting constitutes what Horn (1968) termed a *generalized solution aid.* A generalized solution aid is a specialized crystallized skill or "technique which may compensate for limitations in anlage capacities" (Horn, 1968, p. 244). Like other crystallized skills, counting may enable the child to solve intellectual problems that are unsolvable given his or her fluid ability. At an elementary level, arrays too large to be within subitizing range may be apprehended through counting, or counting may be used to compare two arrays that because of deceptive length and density cues are not easily evaluated by simple inspection. By contrast, Genevan theory implies that counting could not compensate for immaturity in operative level. Operative level would enjoy a monarchic relation to counting. For example, the child's use of counting to read off the cardinal value of an array would not ensure the child's understanding of number invariance. For Piaget, such a "reading off" would constitute a false conservation. Available evidence suggests that accurate counting, although an excellent means of determining the cardinal value of an array, does not guarantee the attainment of conservation of number (Carpenter, 1971; Gréco, 1962; Piaget, 1965; Wallach & Sprott, 1964; Williams, 1971; Wohlwill & Lowe, 1962; Zimilies, 1966). Yet once conservation of number is attained, the child may use the cardinal values he or she abstracts from arrays in explaining the invariance property.

The set of tasks used in the present study are abridged versions of correspondence tasks used by the investigator in another study (Schonfeld, 1982). The tasks involve items in which accurate comparisons require the coordination of length and density cues, two- and none-to-one correspondences, and countervailing subarray (greater + lesser) relations. The relations embodied in these tasks include compensatory relations as well as the injective and surjective correspondence relations described by Piaget et al. (1971). Performance on the tasks is hypothesized to reflect the level of the child's understanding of compensatory and correspondence relations (Schonfeld, 1982). An implication of the Genevan theory of functions and correspondences (Piaget, 1968, 1970a, 1970b, 1970c; Piaget, Grize, Szeminska, & Vinh Bang, 1971) is that the types of correspondence relations embodied in the tasks used in the present study are more readily solved by concrete operational thinkers than by preoperational thinkers. Genevan theory holds that because learning is regulated by operative level the child's effectiveness in applying solution aids such as counting and matching is limited by the child's level of operative intelligence. However, the "new look" in Genevan theory embodied in the work on function and correspondence relations (Piaget et al., 1977) that has emerged in the 1960s includes the attribution of positive cognitive accomplishments to preoperational children. The Genevan theory of functions would distinguish between preoperational children who have attained some capacity to understand correspondence relations (e.g., some preoperational children have mastered the ingredients of the one-to-one correspondence) from those who have not.

Cattell-Horn theory, however, implies that the implementation of solution aids should enhance the performance of the preoperational child provided that the solution aids are within the child's knowledge base. Consistent with the Cattell-Horn position, there has been mounting evidence that children as young as three are capable of representing numerosity through counting and correspondence (Gelman & Gallistel, 1978). It is therefore important that the investigator include only those children who show evidence of mastery of the required skills. The tasks employed in the present study were designed such that the solution aids of counting and matching may be readily applied.

Because counting becomes routinized and systematic with development, it constitutes a cognitively efficient (Beilin, 1969) means of making numerical comparisons. Matching also constitutes a solution aid, albeit a less efficient one. Matching as it is used here means that the child uses his or her finger to create pairs of objects belonging to facing arrays. The child does externally with his or her finger what, otherwise, he or she might do mentally. If a child matches, one-to-one, pairs of facing elements (for example, in the first LD array pair in Figure 1) from two arrays, proceeding from left to right, as soon as he or she finds a member of one array without a correspondent in the other array, the array containing an unmatched element may be judged to be the more numerous. Matching, in comparison to counting, is probably subject to less intense acculturational pressures.

In the present study, the effectiveness of counting and matching as solution aids was contrasted to that of inspection. Of particular interest is the relative effectiveness of solution aids within a sample of preoperational children. In a prior study (Schonfeld, 1982) children of different developmental levels inspected pairs of arrays embodying different correspondence relations. The use of an instructional set inhibited the children from using overt counting or matching in obtaining solutions. Operative level was found to be highly related to performance on the four most difficult correspondence tasks, those from which the tasks used in the present study were derived. One purpose of this study is to investigate the facilitative effects of solution aids on the comparison-making capabilities of preoperational children, thus allowing for a comparison of predictions made by Genevan and Cattell-Horn theories. A prediction that follows from the Genevan viewpoint is that operative level structures children's capacity to use the solution aids to compare arrays accurately (an interaction in statistical terms). Relative to the performance of peers who make numerical comparisons by inspecting arrays, the use of solution aids (counting or matching) should minimally enhance the performance of the most primitive preoperational children; however, at more advanced operative levels the use of solution aids should enhance performance considerably, relative to inspection. By contrast, a prediction that follows from the Cattell-Horn viewpoint is that the solution aids, provided they are part of the child's repertoire of cognitive skills, ought to enhance the performance of all children including that of the most primitive preoperational children (no interaction).

It should be noted that this study also addresses a methodological similarity between the Genevan and Cattell-Horn theories. The originators of the theory of fluid and crystallized intelligence and the theory of operatory intelligence employed nonexperimental methodologies in advancing their theoretical goals (cf. Elkind, 1974). Although many American researchers have subjected Genevan theory to experimental scrutiny, this researcher finds that few investigators have subjected Cattell-Horn theory to similar scrutiny. The present study, however, asserts that experimental scrutiny of both theories is useful.

Method

Subjects

Each of 105 children who ranged from 4 years, 0 months to 7 years, 11 months was examined. The mean age was 5 years, 11 months. Children attended tuition-charging private schools, and an informal review of parental occupations indicated that parents were generally employed as professionals and business people. Both sexes were about equally represented. Approximately 90% of the children were white.

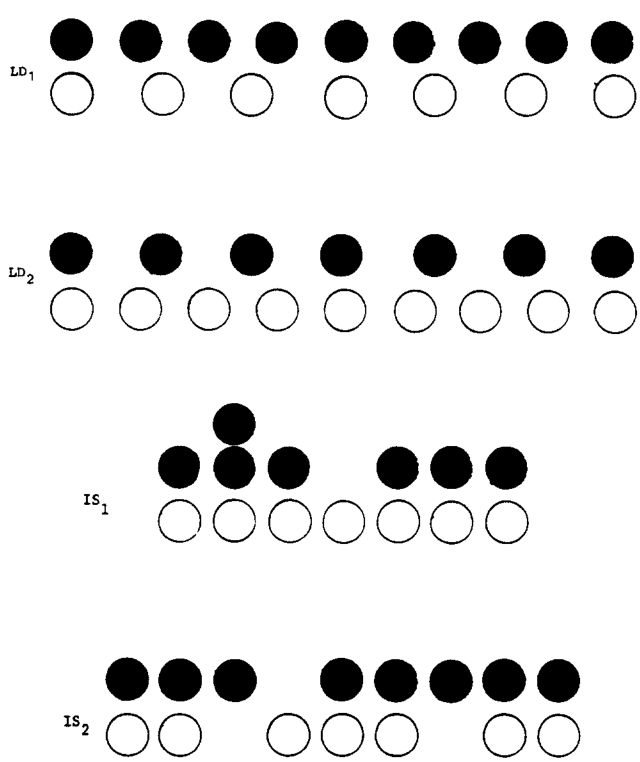

Figure 1.

Materials

A Sesame Street finger puppet named Grover and 10 black checkers were used in the counting task. Eight black checkers and eight red checkers were used in the conservation of number task. Sesame Street finger puppets named Bert and Ernie were used in the static numerical comparison tasks and the instructional set conditions. Each pair of arrays used in the comparison tasks and the instructional set conditions consisted of a row of green and a row of red decals that had been pasted on to a 15 in. × 4 in. (38.10 cm × 10.16 cm) white rectangular cardboard surface. Each decal was circular with a diameter of three-quarters of an inch (1.9 cm).

The arrays used in the static numerical comparison tasks ranged from 7 to 10 in number. Red decals always appeared above green. Examples of the arrays are depicted in Figures 1 and 2. Each of the four static numerical comparison tasks, LD, IS, TP-I, TP-S, involved four different pairs of arrays. The four arrays used in any one task had a common feature to elicit the child's knowledge of compensatory relations. In the LD task (for length-density compensation from Beilin's [1969] test for "conservation of inequality") the members of a pair of arrays were unequal in number, different but uniform in density, and aligned at the terminal decals. In the IS (injective-surjective correspondence) task, paired arrays were irregularly matched one-to-one, none-to-one, and two-to-one, with terminal decals aligned. The TP-I and -S (two part -injective and -surjective) tasks involved arrays that were themselves divided into two subarrays. In the injective version of the TP tasks, one- and none-to-one mappings were employed. In the surjective version, one- and two-to-one mappings were employed. In all but one of the eight pairs of arrays used in the two TP tasks, terminal decals were aligned. In each of the TP tasks subarray relations were reversed. That is to say, every pair of TP arrays was structured such that: (a) of the two subarrays on the left, a subarray of one color had a greater number of decals than the corresponding subarray of the other color (for example, there might be more green than red decals on the left); and (b) of the two subarrays on the right, the color of the subarray that was the more numerous was the same as the color of the subarray that on the left was the less numerous (and thus more red than green decals on the right).

In contrast to the arrays used in the static numerical comparison tasks, the three red–green pairs of practice arrays employed in the instructional set conditions were smaller and less complex. The following pairs of arrays were employed: 5 red versus 2 green; 3 red versus 6 green; and 4 red versus 4 green. The rows of decals making up a pair of practice arrays were linear, equally dense, and, as far as possible, matched one-to-one from left to right. The red decals always appeared above the green.

Design

Participating children were seen twice no more than four days apart. At the beginning of the first session, each child was asked to count 10 checkers. Any child who counted inaccurately was not included in the sample. Approximately half of the children who were included in the sample were administered four static numerical comparison tasks during the first session. Four other tasks, which were part of a coordinate, but separately reported, study on children's capacity to compare liquid quantity (Schonfeld, 1982), were administered during the second session. For these children Session 1 consisted of, in order, the following: a conservation of number test, one of three possible instructional sets, four static numerical comparison tasks, and a re-presentation of the number conservation test. The purpose of the second number conservation test was to assess the acquisition of number conservation as a consequence of experience with the static numerical comparison tasks.

The other half of the children were administered the static numerical comparison tasks in Session 2; however, the instructional set was always

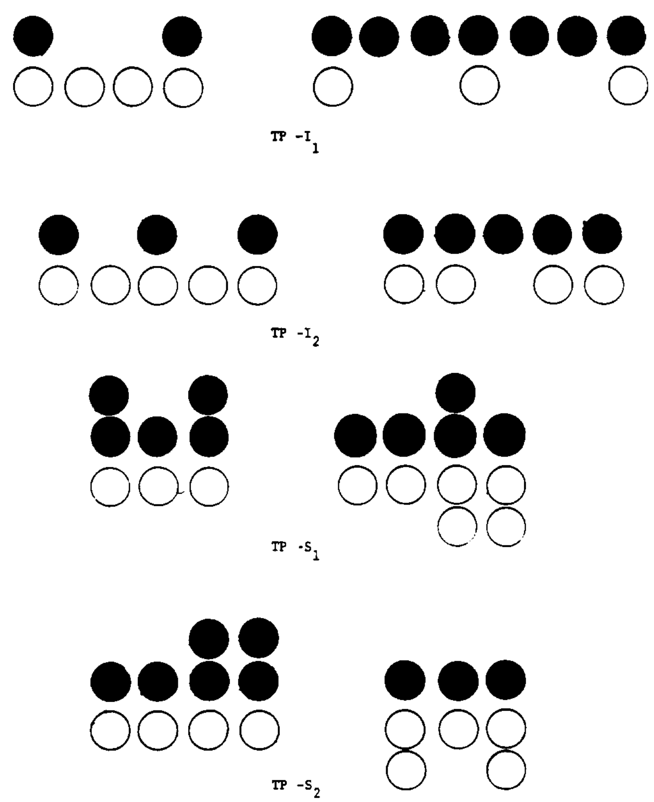

Figure 2.

administered at the outset of Session 1 just after the counting task [and before the liquid tasks]. For these children Session 2 consisted of, in order, the following: a number conservation test, a reminder of the instructional set administered in Session 1, four static numerical comparison tasks, and a re-presentation of the number conservation test. The order in which the comparison tasks were administered was counterbalanced for subgrouping (unitary arrays vs. arrays comprising subarrays). Thus in either session, subjects were administered numerical tasks in one of the following two orders: (a) number conservation (NC), LD, IS, TP-I, TP-S, NC; (b) NC, TP-I, TP-S, LD, IS, NC.

Procedure

Counting task. Each child was introduced to the Grover puppet. The child was then shown a row of 10 black checkers. The examiner informed the child that the checkers were Grover's and asked the child to help Grover find out how many the puppet had by counting the checkers. Two children counted inaccurately and were excluded from the sample.

Instructional set. Children were randomly assigned to one of three instructional set conditions, inspection, counting, or matching. Within each set condition, the child was introduced to the Bert and Ernie puppets and was told that each puppet received candy—Bert received green candy and Ernie, red candy—from his mother. Three pairs of arrays of red and green decals, the practice arrays described above, were represented as the candy. The arrays were deliberately kept small in size in order to insure that the set was easy to acquire. Each child assigned to the inspection set condition was instructed to compare the members of each pair of arrays by careful inspection, the way the puppets, who hadn't yet learned to count, preferred the comparison to be made. The child was asked if Bert had more candy, if Ernie had more candy, or if both puppets had the same amount of candy.

The counting set condition was identical to the inspection set condition except that each child was told that Bert and Ernie liked to count and preferred that counting be used to compare the green and red candies in the practice arrays. The matching set condition was identical to the other two conditions except that the child assigned to the condition was told that Bert and Ernie liked to use a finger to match, and preferred that one-to-one matching of the green and red candies be used to compare the practice arrays. Any child who did not understand the set instructions was briefly shown how to perform in accordance with the instructions. This was rare; however, a small number of the younger children in the matching condition required one demonstration.

When the children were administered the static numerical comparison tasks in the second session, the children were reminded of the set instructions administered at the beginning of Session 1. The examiner showed the child the Bert and Ernie puppets and asked the child how the puppets compared candies. If the child responded incorrectly, the examiner repeated the set instructions.

Conservation of number test. The number conservation test was administered before and after the static numerical comparison tasks. The test consisted of three trials. In the first trial eight red (black) checkers were placed in a row before the child who was then asked to remove from a bag as many black (red) checkers. The examiner recorded whether or not the child placed the two sets of checkers in one-to-one correspondence. Once the one-to-one correspondence was established (either by the child or the examiner) the examiner expanded (compressed) the row of black (red) checkers and asked the child if the two rows still had the same number of checkers or if one row had more. The child was then asked the reason for his or her judgment. The second trial began after the examiner reestablished the one-to-one correspondence. The examiner compressed (expanded) the red (black) row and then questioned the child. The orders of the first two trials and the colors of the checkers in the rows to be transformed were counterbalanced. The third trial paralleled the second except that the red (black) row was stacked to form a cylinder.

Children were poststratified according to three operative levels: Level 1 nonconservers (most primitive), Level 2 nonconservers (intermediate), and conservers (most advanced). Children who failed to respond correctly on all pre- and posttest trials and failed to place the red and black checkers in one-to-one correspondence on at least one test were operationally defined as Level 1 nonconservers (Level 1 NCs). Children who failed to respond correctly on all pre- and posttest trials but placed the red and black checkers in one-to-one correspondence on both tests were considered to be more advanced and were operationally defined as Level 2 nonconservers (Level 2 NCs). Children who, on each number conservation test, responded correctly and supplied adequate justifications for their responses (e.g., reversibility, addition/substraction) on at least two of the three trials were operationally defined as conservers of number. In Piagetian terms, each of the two NC groups was considered to be preoperational and the conserver group, concrete operational. One child who evinced mastery of one-to-one correspondence on both conservation tests and responded correctly on all pre- and posttest trials without supplying an adequate justification for his responses was classified, in Piagetian terms, as transitional. Because there was only one transitional child, the category was excluded from the analysis. If any child responded incorrectly on all pretest trials but correctly, and with justification, on at least two of three posttest trials, he or she would have been operationally defined as an improver. No child whose behavior conformed to this pattern was found.

Static numerical comparison tasks. Every child was administered four static numerical comparison tasks, the LD, IS, TP-I, and TP-S tasks. Each task consisted of four comparisons involving the paired arrays described in the materials section. In each task the child compared the green decals, represented as the Bert puppet's candy, to the red decals, represented as the Ernie puppet's candy. During every comparison the Bert puppet stood next to the row of green decals and the Ernie puppet, next to the row of red decals. With the presentation of each array pair in the LD and IS tasks the child was asked "Did Bert get more candy? Or did Ernie get more candy? Or did both puppets get the same amount of candy?" The orders in which the questions were asked were rotated.

The administration of the two-part (TP) tasks differed, particularly in the beginning of each task, from the administration of the LD and IS tasks. With the presentation of the first array pair in each of the TP tasks, the subarrays on the left were presented as the candy the puppets received in the morning and the subarrays on the right, as the candy the puppets received in the afternoon. With a sweeping motion the examiner indicated that each row of decals across the entire card was the candy each puppet got for the whole day. The examiner next asked the child to compare the morning candies and then the afternoon candies. Every child made these comparisons accurately. The practice of comparing morning and afternoon subarrays was not continued for the remaining three comparisons within each of the TP tasks. It was evident from these and from pilot data that children make subarray comparisons accurately. To elicit the child's comparison of the first entire array pair, the child was asked to compare the candies the puppets got "for the whole day, morning and afternoon together." For each of the next three comparisons within each TP task, the child was asked a set of questions paralleling the questions asked in the LD and IS tasks. The only difference was that the questions used in the TP tasks were prefaced by the phrase "for the whole day."

Throughout the administration of the static numerical comparison tasks the examiner monitored the child's responses to determine if the child's behavior conformed to the instructional set. Although this was rarely needed, any child whose behavior failed to conform to the instructional set was prompted to inspect, count, or match accordingly.

Results

In order to maximize reliable variance in the dependent measures and thus increase the power of the tests to be conducted, items from the tasks were pooled to constitute two scales. Scale 1, reflecting performance on comparisons involving unitary ar-

Table 1
Zero-Order Correlations: Scales 1 and 2, Operative Level, and Age

	1	2	3
1. Scale 1			
2. Scale 2	.68*		
3. Operative level	.72*	.54*	
4. Age	.75*	.56*	.73*

* $p < .001$.

Table 2
Numbers of Subjects in Each Cell

Operative level (poststratification factor)	Inspection	Set counting	Matching	Row sum
Level 1 NC	8	8	7	23
Level 2 NC	12	13	14	39
Conserver	15	10	18	43
Column sum	35	31	39	105

rays, consisted of the eight items from the LD and IS tasks. Scale 1 thus constituted a measure of the child's understanding of compensatory relations uncomplicated by the presence of two-part comparisons. The KR-20 reliability (Cronbach's alpha for scales in which response choices are scored as correct or incorrect) for Scale 1 was .86. Scale 2, reflecting performance on comparisons involving arrays comprising subarrays, consisted of the eight items from the two TP tasks. Scale 2 thus constituted a measure of the child's understanding of complex two-part compensatory relations. The KR-20 reliability for Scale 2 was .77.

Table 1 presents zero-order correlations between pairs of the following variables: Scale 1, Scale 2, operative level, and age. The correlation between the two scales approximates the limiting value permitted by the reliabilities. Both scales were moderately correlated with age and operative level. Age and operative level were also moderately correlated. Operative level was significantly correlated to Scale 1 ($r = .39$, $p < .001$) and Scale 2 ($r = .23$, $p < .05$) when age was partialed.

With performance on the number conservation tests as an index of operative level, it was expected that, from the Genevan standpoint, set would minimally influence the performance of the least advanced preoperational children (Level 1 NCs) and that the performance of the conservers or the Level 2 NCs would be most influenced by the introduction of solution aids. By constrast, Cattell-Horn theory implies that if a Level 1 NC could count accurately, the use of counting as a solution aid should enhance performance on the static numerical comparison tasks. With regard to the performance of Level 2 NCs and conservers, Cattell-Horn theory does not conflict with Genevan theory because both would suggest that solution aids are useful and accessible to those at higher operative levels. Thus a key test of the Cattell-Horn and Genevan viewpoints would be found in the presence or absence of an interaction.

In order to test the hypothesis a two-way operative level (3 levels) by set (3 levels) analysis of variance (ANOVA) was conducted using each scale as a dependent measure. Because of unequal cell size owing to the poststratification by operative level (see Table 2) all effects were assessed using a regression approach (Cohen & Cohen, 1983). The mean scale scores at each operative level cross-classified by set condition are presented in Table 3. Main effects on Scale 1 were found for operative level, $F(2, 96) = 70.62$, $p < .001$, and set $F(2, 96) = 6.01$, $p < .01$. An operative level by set interaction, $F(4, 96) = 2.59$, $p < .05$, was also detected. Scheffé post hoc tests indicated that the mean Scale 1 score for conservers differed significantly from that of Level 1 and Level 2 NCs ($p < .01$) and that the mean Scale 1 score for Level 2 NCs differed significantly from that of Level 1 NCs ($p < .01$) [nonsignificant differences are not presented]. Post hoc tests also indicated that subjects assigned to the counting condition performed significantly better than subjects assigned to the inspection condition ($p < .01$).

Main effects on Scale 2 were found for operative level, $F(2, 96) = 28.91$, $p < .001$, and set, $F(2, 96) = 8.16$, $p < .001$. Although the patterning of the Scale 2 cell means suggests counting helps conservers more than nonconservers, no significant operative level by set interaction was detected. Scheffé post hoc tests indicated that the mean Scale 2 score for conservers differed significantly from that of Level 1 and 2 NCs ($p < .01$). Post hoc tests also indicated that children assigned to the counting condition performed significantly better than children assigned to the inspection condition ($p < .01$).

In order to locate the source of the interaction in the assessment of the children's performance on Scale 1, simple one-way effects (Winer, 1971) were assessed for set within each operative level. Set effects were found to be significant only within the Level 2 NC group, $F(2, 96) = 11.16$, $p < .001$. Scheffé post hoc tests indicated that performance of Level 2 NCs assigned to the counting set was significantly ($p < .05$) better than that of peers assigned to either inspection or matching. Simple one-way set effects were absent within the conserver and Level 1 NC groups. Simple one-way effects were also assessed for performance on Scale 2. Significant set effects were found within the conserver group, $F(2, 96) = 3.97$, $p < .05$; marginally significant set effects were found within the Level 2 NC group, $F(2, 96) = 2.55$, $p < .10$; and no set effects were found within the Level 1 NC group.

Table 3
Mean Scale Score by Operative Level and Set Condition

Operative level	Set: Inspection	Set: Counting	Set: Matching	OL mean
		Scale 1[a]		
Level 1 NC	1.75	1.50	1.14	1.48
Level 2 NC	1.83	5.23	3.43	3.54
Conserver	6.07	7.60	6.67	6.67
Set mean	3.63	5.03	4.51	
		Scale 2[a]		
Level 1 NC	1.62	2.75	2.71	2.35
Level 2 NC	2.83	4.62	2.71	3.38
Conserver	4.53	7.30	5.50	5.58
Set mean	3.29	5.00	4.00	

[a] Maximum scale score = 8.

Additional analyses were conducted in order to determine if the operative level main effects and the operative level by set interaction would persist when age was covaried. An operative level by set analysis of covariance (ANCOVA) was conducted after a test, on each scale, for the viability of the homogeneity of slope hypothesis failed to be rejected. With regard to Scale 1, the ANCOVA revealed an operative level main effect, $F(2, 95) = 13.96$, $p < .001$, a set main effect, $F(2, 95) = 6.88$, $p < .01$, and an operative level by set interaction, $F(4, 95) = 7.00$, $p < .005$, when age was covaried. To highlight the source of the interaction, Table 4 presents the age-adjusted deviations between the cell means and the unweighted row means for Scales 1 and 2. The adjusted mean deviation, because it is a measure of difference, is a better indicator of effect than the adjusted cell mean, which is not a true group average (Shrout, personal communication, 1985). Table 4 indicates that, consistent with the results of the ANOVA, the solution aids, particularly counting, tended to affect performance most in the Level 2 NCs.

With regard to Scale 2, the ANCOVA revealed operative level, $F(2, 95) = 4.84$, $p < .01$, and set, $F(2, 95) = 8.37$, $p < .001$, main effects when age was covaried. Although Table 4 is suggestive of an interaction effect as reflected in the age-adjusted deviations found in the conservers (in comparison to the NCs), such a conclusion is not warranted. The interaction effect for Scale 2 was not significant ($p < .25$).

Table 4
Age-Adjusted Deviations Between the Cell and the Unweighted Row Means

Operative level	Set		
	Inspection	Counting	Matching
	Scale 1		
Level 1 NC	0.35	−0.03	−0.32
Level 2 NC	−1.47	1.53	−0.07
Conserver	−0.75	1.03	−0.27
	Scale 2		
Level 1 NC	−0.70	0.34	0.35
Level 2 NC	−0.41	1.08	−0.67
Conserver	−1.27	1.66	−0.38

Discussion

The study investigated the relation of operative level and set to performance on tasks indexing children's knowledge of correspondence relations. Each of the two scales reflecting subject mastery of correspondence relations was found to have satisfactory internal consistency reliability. Performance on both scales was moderately correlated to age. Operative level effects on performance on both scales were found. The operative level effect remained when age was partialed or covaried. As expected, set effects indicating that counting is a better solution strategy than inspection were found. Children assigned to the matching condition tended to perform at levels intermediate to those found in the inspection and counting conditions. An operative level by set interaction was found for Scale 1; however, no interaction was found for Scale 2.

The results tended to be consistent with the Genevan view in that operative level (a) was closely tied to performance on the comparison tasks and (b) moderated set effects on Scale 1. An alternative explanation for the findings is that, like many aspects of cognitive development (including vocabulary acquisition and abstract reasoning), both operative level and correspondence-based understanding are related to age; therefore, the finding of a relation between operative level and correspondence-based knowledge is not especially supportive of Genevan theory. In response to such an explanation it should be noted that operative level was found to be significantly related to performance on Scales 1 and 2 when age was statistically controlled. The operative level by set interaction for Scale 1 remained when age was covaried.

The presence of an interaction effect is important because it is consistent with the Genevan view that operative level regulates the knowledge-producing capabilities of the solution aids. This view is supported by the results pertaining to Scale 1. The results that apply to Scale 2, however, are more equivocal. Although the operative level by set interaction failed to attain statistical significance, simple one-way ANOVAs were either significant or marginally significant for conservers and Level 2 NCs.

Performance on Scale 2 differs from performance on Scale 1. The conservers tended to perform better on Scale 1 and the level 1 NCs, better on Scale 2. The finding pertaining to conservers is in keeping with the notion that Scale 2 captures more complex compensatory relations and, therefore, is more difficult. Why, however, should the Level 1 NCs assigned to the counting and matching conditions obtain higher mean scores on Scale 2 than on Scale 1? One possible explanation is that the Scale 2 items elicited a higher level of guessing or random responding in the subjects assigned to the counting and matching conditions than did Scale 1. This explanation is consistent with the finding that the cell means for subjects assigned to counting and matching were very close to the value to be expected if all subjects were to arrive at judgments randomly (2.67). An implication of this possibility is that Level 1 NCs who performed worse than chance on Scale 1 made systematic errors. Perhaps they used very immature, but systematic, comparison strategies. A study to identify such strategies is needed to elucidate the issue. It is, however, unlikely that almost every Level 1 NC assigned to the counting or matching condition guessed in response to each Scale 2 item. Perhaps these subjects implemented a variety of strategies, undetectable given the present methods, that when aggregated yielded results resembling a random process.

It is suggested here that the quality of the comparison tasks and the level of the child's thinking within the preoperational period are key issues surrounding the question of the cognitive achievements of preoperational children. The tasks employed in the present study were designed to embody correspondence relations specified by late Genevan theory. A special concern of the present article was to develop tasks whose psychometric properties allowed for adequate tests of the hypotheses; the psychometric properties of experimental tasks employed in cognitive developmental research are too infrequently investigated (Rushton, Brainerd, & Pressley, 1983).

With these concerns in mind, the issue studied here was not whether preoperational children can achieve success on certain tasks, but the identification of levels within a heterogeneous preoperational period that allow for the use of effective solution procedures. In the present study, preoperational children were grouped by two levels: those who manifested little spontaneous capacity to use one-to-one correspondence in establishing quantitative equivalence (Level 1 NCs) and those who easily used one-to-one correspondence in establishing equivalence (Level 2 NCs). Level 2 NCs assigned to the counting condition, in comparison to their developmental peers assigned to the inspection condition, tended to be more accurate in comparing Scale 1 arrays. This counting-inspection differential did not materialize for Level 1 NCs. It might be argued that these findings may be owed to the presence of more accurate counters among the Level 2 NCs than among the Level 1 NCs. Such an explanation, however, is inconsistent with a key feature of the experimental procedure. In order to be incepted into the study subjects had to pass a test of counting accuracy geared to the largest numerosity encountered in the scale items. Thus the differences favoring counting over inspection for Level 2 compared to Level 1 NCs could not be attributed to operative-level-related differences in counting accuracy, at least as far as numerosities of 10 or less are concerned. Moreover, direct observation during the tasks indicates that the Level 1 NCs tended to count accurately. For example, a Level 1 NC might accurately count seven red and nine green candies in an LD comparison but indicate that the two puppets received the same amount.

The results are consistent with the view that operative level moderates, or structures, the child's capacity to use solution aids. One possible explanation is that, in contrast to Level 1 NCs, Level 2 NCs have, as evidenced by their performance on the number conservation tests, a greater understanding of one-to-one correspondence relations. Level 2 NCs would thus be more likely to understand the significance of counting, which itself involves the establishing of a one-to-one correspondence relation between an ordered list of number names and a set of countables (see Saxe, 1979a, 1979b). It would, however, be useful for future researchers to develop reliably differentiated categories within the preoperational period in order to explore further the issue of operative-level-related dependencies in functioning. The above findings are limited to Scale 1. Continued scale development would be useful.

Despite the support offered by this article for the Genevan view, Genevan theory, with its stress on universal features of cognitive development, tends to neglect issues pertaining to localized functioning such as the question of the relative efficacy of rival solution approaches. Cattell-Horn theory, by contrast, emphasizes that well-learned knowledge-producing skills constitute potential solution aids, and that children differ in what solution aids they learn to implement. That children assigned to the counting condition tended to make more accurate comparisons than children assigned to the two other conditions implies that counting constitutes a solution aid that is cognitively more efficient (Beilin, 1969) than either inspection or matching. Several characteristics of counting are possible sources of efficiency. Counting is an indexing operation where number names are ordered and assigned to objects one-to-one, thus guaranteeing an accurate representation of the cardinal value of an aggregate. Counting becomes highly routinized and thus easy to invoke. According to Werner (1957), counting, with development, becomes progressively less susceptible to interference related to the configuration of countables. Finally, counting is a "tool" with strong environmental support (Saxe, 1979b; Vygotsky, 1978).

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Received February 22, 1984
Revision received July 1, 1985 ■