Developmental Psychology
1990, Vol. 26, No. 1, 94–102

0012-1649/90/$00.75

The Child's Understanding of Correspondence Relations

Irvin Sam Schonfeld
Department of Social and Psychological Foundations
City College of New York

A number of quantitative comparison tasks were designed to tap knowledge of injective and surjective correspondences, one-directional compositions (greater + greater yields greater), countervailing compositions (greater + lesser yields ?), and length-density relations in 4- to 7-year-olds. The results indicated that performance on the comparison tasks was related to performance on a number conservation test as well as to age. Nonconservers performed at better than chance levels on tasks that tapped an elementary knowledge of injective and surjective correspondences; concrete-operational children, however, tended perform better on all tasks. Uncorrected and disattenuated correlation coefficients revealed considerable consistency across measures. Factor analyses, with and without age included, yielded a unitary factor. An explanation of the results based on perceptual salience was ruled out.

In the 1960s a "new look" in Genevan theory emerged with Piaget's description of a psychology of functions and correspondences. An important feature of the theory has been the attribution of a number of cognitive accomplishments to preoperational children. Previously, the Genevans were more apt to characterize preoperational children by the cognitive behaviors they lacked in comparison with concrete-operational children (Schonfeld, 1986). Developments in the Genevan psychology of functions and correspondences have enabled investigators to obtain more detailed descriptions of what both preoperational and concrete-operational children can accomplish (Piaget, Grize, Szeminska, & Vinh-Bang, 1968/1977).

The development of a psychology of functions and correspondences depends on operationalizing tasks to assess the conceptual understanding of children (Davidson, 1988). In this study specific correspondence relations described by Piaget et al. (1968/1977) were operationally defined in the context of quantitative comparisons. Quantitative comparisons were chosen because cognition about quantity plays an important role in Genevan theory (Piaget, 1941/1965), and correspondence relations are easily embedded in quantitative comparisons.

On the basis of the new look, the child's understanding of two types of correspondence relations, injection and surjection, was examined. In the injective correspondence every element in a set B corresponds to, at most, one element in a set A (Piaget, 1977; Piaget et al., 1968/1977). In other words, injection entails a relation in which every element in B corresponds to one or no element in A. By contrast, in the surjective correspondence, every element in set B corresponds to one or more elements in set A. In both injection and surjection, every element in A corresponds to exactly one element in B, but not vice versa. Injection and surjection constitute "one-way" functions. Examples of injective and surjective correspondences are depicted in Figure 1. In the pair of arrays labeled I_1, every light dot corresponds to exactly one dark dot. Not every dark dot, however, corresponds to a light dot. In the pair of arrays labeled S_2, every dark dot corresponds to exactly one light dot. The reverse is not true.

The injective and surjective comparisons to be examined differ from classic conservation comparisons. The quantities involved are static. That is, the child does not witness the transformation of quantities, as is the case in classic conservation tests. The investigation of children's knowledge of untransformed, or static, quantities is as important as the investigation of their knowledge of transformed quantities, because many everyday comparisons involve quantities in which no transformation is involved (Beilin, 1969). Moreover, such an investigation is in keeping with what has been a departure for the Genevans, who had formerly conceptualized knowledge mainly in terms of "transformations," but who, more recently, hypothesized an important role for correspondence relations in comparisons between static states (Piaget, 1977). Although investigators have studied the child's capacity to compare untransformed quantities (e.g., Beilin, 1969; Brainerd, 1977; Cowan, 1984, 1987a, 1987b; Fuson, 1988; Michie, 1984a, 1984b; Pufall & Shaw, 1972; Pufall, Shaw, & Syrdal-Lasky, 1973; Saxe, 1977; Schwartz & Scholnick, 1970; Zimiles, 1966), little work on quantitative comparisons has been done bearing on the Genevan view of correspondences. On the other hand, there has been increasing interest in examining the psychology of functions and correspondences in other domains (e.g., Davidson, 1987; Dean & Deist, 1980).

Evidence adduced by several investigators (Brainerd, 1977;

This article is based on a substantive reanalysis of data collected in one of the studies conducted for my doctoral dissertation. The dissertation was submitted to the City University of New York in partial fulfillment of the requirements of the PhD degree.

I thank the following individuals for their critical reading of earlier versions of the article: Harry Beilin, Geoffrey Saxe, and three anonymous reviewers. Special thanks are extended to Alan Gross, Bennett Gewirtz, and Pearl Knopf.

Correspondence concerning this article should be addressed to Irvin Sam Schonfeld, EDFN, City College, New York, New York 10031.

Piaget, 1968/1970; Pufall & Shaw, 1972; Pufall et al., 1973) suggests that young children master at least two types of static comparisons. One type involves arrays that are the same length and number. The other involves pairs of arrays in which the longer one is the more numerous. According to the Genevans (Piaget et al., 1968/1977), these types of comparisons require the mapping of spatial extent schemata onto schemata representing numerosity. The preoperational child, in applying spatial extent schemata to compare arrays in which relative numerosity and spatial extent conflict (e.g., the length–density arrays depicted in Figure 2), is bound to err (Beilin, 1969). In contrast, 4-year-olds in a study conducted by Brainerd (1977) appeared to judge relatively accurately, and precociously, same-length/different-number type problems, although some uncertainty surrounds the results. The correct response was confounded with the color (black) of the more numerous array (Cards 7 and 8) and, among all the inequality comparisons, black dots outnumbered red.

In the study described here, performance on the correspondence tasks is linked to performance on the traditional conservation of number test. It should be remembered that, in his later writings, Piaget (1967/1971) minimized the role of the stage concept and specifically emphasized continuity in cognitive development. The evolution from "preoperational" to "concrete-operational" thought is essentially a continuous process. Performance on a number conservation test is one marker of the child's level of cognitive development (Hooper, Fitzgerald, & Papalia, 1971) and provides well-recognized, if not the only, landmarks in the continuous development of the child's understanding of the number concept. The test was, therefore, used to independently identify preoperational children, operationally defined by their nonconserving status, in order to highlight their cognitive accomplishments.

Piaget (1977) advanced the view that knowledge of correspondences arises out of the application of primitive action schemes that "do not transform objects to be compared but that extract common forms from them or analogies between them" (p. 351). Genevan theory suggests that preoperational children would, with some accuracy, compare the relative quantity of two arrays embodying elementary injective or surjective correspondences. Static-state differences between the arrays require only a one-way ordering of the pair by the relative excess of gaps or circles in one of them. This study examines the performance, on elementary injective and surjective comparison tasks, of preoperational youngsters who were classified as more or less advanced on the basis of their behavior on the one-to-one correspondence component of the number conservation test (Piaget, 1941/1965).

The Genevans also hold that during the concrete-operational period the child's understanding of correspondence relations becomes more elaborated. Compensation, considered by the Genevans to be an emergent feature of concrete-operational thought (see Silverman & Rose, 1982), appears to play a role in the development of the child's understanding of correspondences. Piaget (1941/1965) wrote that compensation includes the capacity to coordinate, or multiply, quantitative dimensions. In the context of the new look, compensation involves the capacity to coordinate functional relations (Piaget et al., 1968/1977). Compensation is relevant to comparisons of arrays that are organized such that each aggregate comprises spatially distinct subarrays (e.g., the two-part one-way, TPO, and two-part reverse, TPR, arrays in Figures 3 and 4). Comparisons of the total arrays call for some cross-referencing, or coordination, of comparisons between subarrays. The following examples illustrate this point.

It is expected that concrete-operational children are more likely than preoperational children to succeed at comparison tasks in which two conditions hold: (a) Visually corresponding subarrays are unequal, and (b) the direction of the inequality that holds between the corresponding subarrays on the left is the reverse of the direction of the inequality that holds between the subarrays on the right (the TPR comparisons in Figure 4). Consider, for example, the two subarrays of the more numerous array in the first TPR comparison illustrated in Figure 4. The left subarrays, from above to below, are R1 and G1 and the right subarrays, R2 and G2 (R represents the red dots and G, the green dots in the comparisons used in this study). Although, from end to end, R is greater than G, R1 is less than G1. This is because the absolute difference between R2 and G2 exceeds the absolute difference between R1 and G1. If R and G were equal, the R1–G1 and R2–G2 differences would exactly compensate for each other (as in the fourth comparison in Figure 4).

Some comparisons involving arrays comprising spatially distinct subarrays are simpler. For example, each subarray of the more numerous array might also be more numerous than the subarray to which it corresponds (the TPO comparisons depicted in Figure 3). In the language of functions and correspondences, the one-way composition of two same-directional subarray relations is needed in making accurate comparisons: $(R1 > G1) + (R2 > G2) \rightarrow (R > G)$. Piaget et al. (1968/1977) hypothesized decalage effects in performance on tasks reflecting the extent to which children understand functions and the composition of functions. It was expected that children should perform better on comparison tasks that involve injective and surjective correspondences embedded in arrays that are not divided into subarrays (Figure 1), because the additional requirement of composing subarray comparisons is not needed. By the same token, it was expected that comparisons that involve the composition of subarrays would be made more easily if the subarray comparisons were in the same direction (as in the TPO arrays depicted in Figure 3) than if the subarray comparisons were countervailing (as in the TPR comparisons depicted in Figure 4).

Another series of arrays was constructed in such a way that the child must coordinate two-, one-, and none-to-one mappings in order to make accurate comparisons (the injective-surjective, or IS, comparisons in Figure 2). It was also expected that compared with concrete-operational children, preoperational children would be relatively inaccurate on IS types of comparisons requiring the coordination countervailing correspondence relations.

Method

Subjects

Sixty-four children, who ranged in age from 4 years, 0 months to 7 years, 5 months, were included in the sample. The mean age was 5 years,

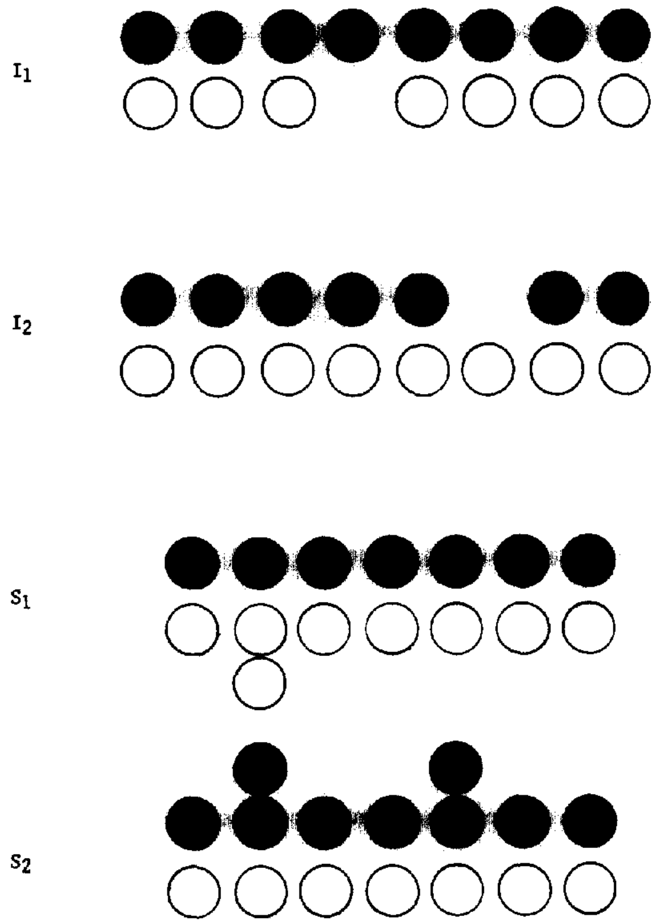

Figure 1. Examples of comparisons in the injective (I) and surjective (S) tasks.

6 months. All children attended tuition-charging private schools. Approximately 90% of the children were White, and a review of parents' occupations indicated that they were professionals and business people.

Materials

Sesame Street finger puppets Bert and Ernie were used in each of the comparison tasks and the instructional set condition. Each pair of arrays used in the comparison tasks and the instructional set condition consisted of a row of green and a row of red decals that had been pasted to a 15-in. × 4-in. (38-cm × 10-cm) white cardboard rectangular surface. Each decal was circular, with a diameter of ¾ in. (1.9 cm). Red decals always appeared above green.

The arrays used in the comparison tasks ranged from 7 to 10 in number. Examples of the arrays are depicted in Figures 1–4. Each of the eight comparison tasks involved seven different pairs of arrays. The seven pairs used in any one task had a common feature to elicit the child's knowledge of correspondence relations. The pairs of arrays in the injective (I) and surjective (S) tasks embodied those correspondences. Two examples of the I and S arrays are depicted in Figure 1. The members of each pair of arrays used in the length–density (LD) task were unequal in number, different but uniform in density, and aligned at the terminal decals. An eighth LD item was mixed into the set of LD comparisons to rule out subject misunderstanding. The item consisted of a pair of arrays that were the same length and density—items were matched one-to-one. In the injective-surjective (IS) task, paired arrays were irregularly matched one-to-one, none-to-one, and two-to-one, with terminal decals aligned. Two examples of the LD and IS arrays are depicted in Figure 2. The two-part one-way subarray composition tasks with injective (TPO-I) or surjective correspondences (TPO-S) and the two-part reversed subarray compositions tasks with injective (TPR-I) or surjective correspondences (TPR-S) involved paired arrays that were themselves divided into two subarrays. A feature of all the TPO pairs was that the color of the more numerous subarray on left was the same as the color of the more numerous subarray on the right, except where a subarray comparison involved equality. Two examples of the TPO-I and TPO-S comparisons are depicted in Figure 3.

In contrast, in the TPR-I and TPR-S tasks, the color of the more numerous subarray on the left was the same as the color of the less numerous subarray on the right. Two examples of the TPR-I and TPR-S comparisons are depicted in Figure 4.

In contrast to the arrays used in the comparison tasks, the three array pairs employed in the inspection set condition were smaller and less complex. The following pairs of arrays were employed: 5 red versus 2 green; 3 red versus 6 green; and 4 red versus 4 green. The rows of decals making up a pair of arrays used in the set condition were linear, equally dense, and, as far as possible, matched one-to-one from left to right. Finally, 8 black and 8 red checkers were used in the test assessing conservation of number.

Design

Each child was initially administered the same instructional set, an inspection set. The purpose of the set was to induce the child to inspect pairs of arrays of "candies" in order to evaluate the relative numerosity

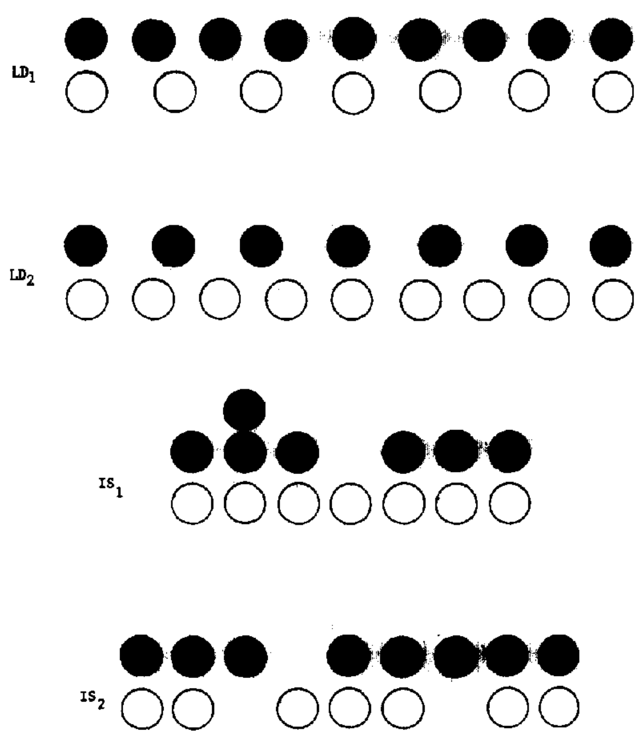

Figure 2. Examples of comparisons in the length–density (LD) and injective–surjective (IS) tasks.

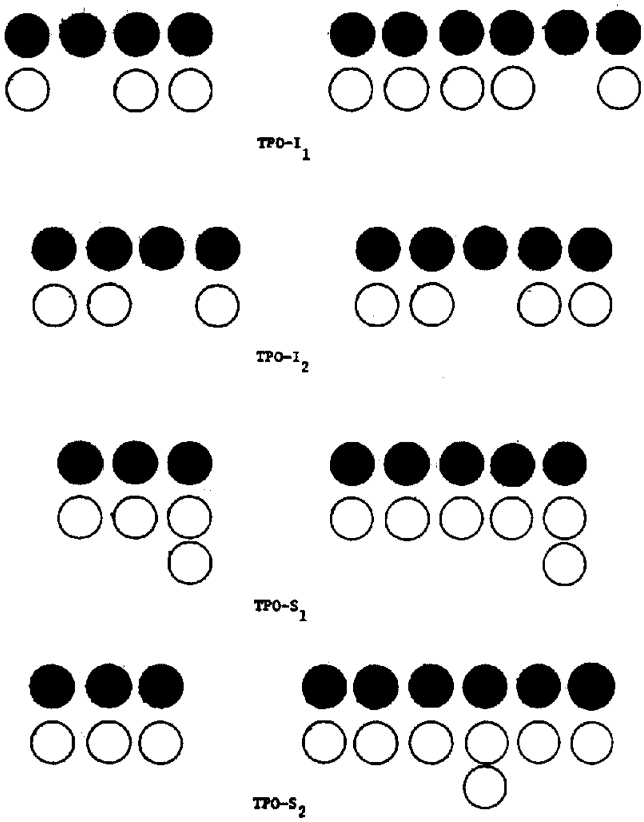

Figure 3. Examples of comparisons in the two-part one-way subarray composition (TPO) tasks.

of the members of each pair. Each child was then administered all the comparison tasks, counterbalanced for subgrouping (unitary arrays vs. arrays comprising subgroups) and hypothesized task difficulty. Four orders of administration were used: (a) I, S, LD, IS, TPO-I, TPO-S, TPR-I, TPR-S; (b) LD, IS, I, S, TPR-I, TPR-S, TPO-I, TPO-S; (c) TPO-I, TPO-S, TPR-I, TPR-S, I, S, LD, IS; and (d) TPR-I, TPR-S, TPO-I, TPO-S, LD, IS, I, S. After the series of comparison tasks was completed, every child was administered a conservation of number test. Performance on the test was used as an independent index of operative level.

Procedure

Inspection set. Every child was introduced to the Bert and Ernie puppets, and was told that each puppet received candy—Bert received green candy and Ernie, red candy—from the puppet's mother. Three pairs of red and green practice arrays described above were represented as candy. The arrays were deliberately kept small in size to ensure that the set was easily acquired. Each child was instructed to compare the members of each pair of arrays by careful inspection, the way the puppets, who had not yet learned to count, preferred the comparison to be made. The child was asked whether Bert had more candy, Ernie had more candy, or both puppets had the same amount of candy. Results from another study (Schonfeld, 1986) in which a variety of instructional sets, including inspection and counting sets, were administered indicate that children's behavior readily conforms to the set instructions (cf. Gibson, 1941; Johnson, 1955; Woodworth, 1937).

Comparison tasks. Every child was administered the eight comparison tasks, the I, S, LD, IS, TPO-I, TPO-S, TPR-I, and TPR-S tasks. Each task consisted of seven comparisons involving the paired arrays described in the Materials section. In each task the child compared the green decals, represented as the Bert puppet's candy, to the red decals, represented as the Ernie puppet's candy. During every comparison the Bert puppet stood next to the row of green decals and the Ernie puppet, next to the row of red decals. With the presentation of each array pair in the I, S, LD, and IS tasks (i.e., the tasks consisting of undivided arrays), the child was asked "Did Bert get more candy? Or did Ernie get more candy? Or did both puppets get the same amount of candy?" The orders of the questions were rotated. The one additional LD comparison in which the two arrays were equal in number was not considered in the subsequent scale construction because every subject indicated that the two puppets had the same amount of candy.

The administration of the two-part (TP) tasks differed, particularly in the beginning of each task, from the administration of the tasks involving undivided arrays. With the presentation of the first pair of arrays in each of the TP tasks, the subarrays on the left were presented as the candy the puppets received in the morning and the subarrays on the right, as the candy the puppets received in the afternoon. With a sweeping motion the examiner indicated that each row of decals across the entire card was the candy each puppet received for the whole day. The examiner next asked the child to compare the candies each puppet received in the morning and then to compare the candies each puppet received in the afternoon. Every child made the subarray comparisons accurately. To elicit a comparison of the entire array, the child was asked to compare the candies the puppets got "for the whole day, morning and afternoon together." The practice of comparing morning and afternoon subarrays was not continued for the remaining six comparisons within each TP task. For the next six within-task comparisons, the child was

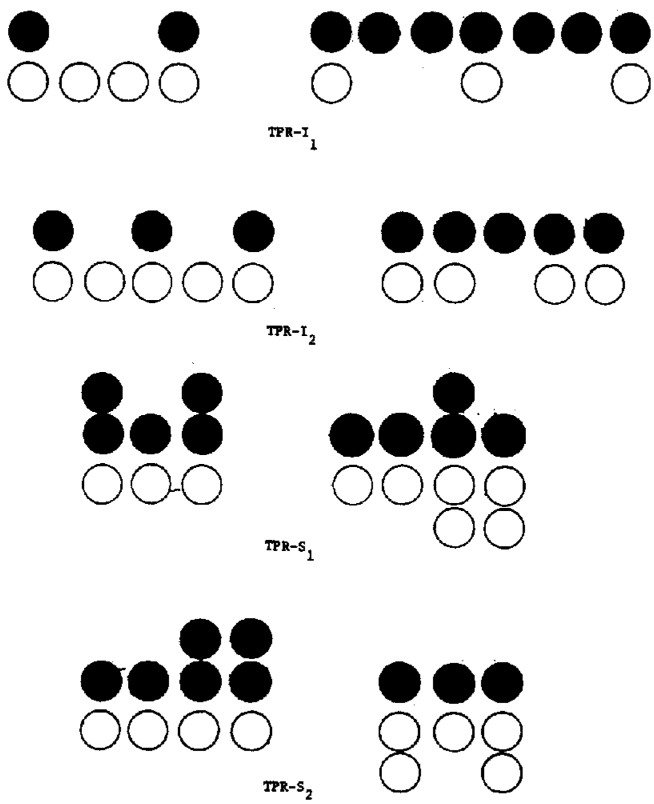

Figure 4. Examples of comparisons in the two-part reverse subarray composition (TPR) tasks.

asked a set of questions paralleling the questions asked in the tasks with undivided arrays. The only difference was that the set of questions asked before each of the next six comparisons was prefaced by the phrase "for the whole day."

Every task but the LD task was constructed such that the correct response was alternated about equally among the following alternatives: Bert having more, Ernie having more, or the two puppets having the same amount. In the LD items, the correct response was never that the two puppets had the same amount; one or the other puppet always had more.

Conservation of number test. The conservation of number test consisted of three trials. In the first trial eight red (black) checkers were placed in a row before the child, who was then asked to remove from a bag as many black (red) checkers. The examiner recorded whether or not the child spontaneously placed the two sets of checkers in one-to-one correspondence. As soon as the one-to-one correspondence was established (either by the child or the examiner), the examiner expanded (compressed) the row of black (red) checkers and asked the child whether the two rows still had the same number of checkers or whether one row had more. The child was then asked the reason for his or her judgment. The second trial began after the examiner reestablished the one-to-one correspondence. The examiner compressed (expanded) the red (black) row and then questioned the child. The orders of the first two trials and colors of the checkers in the rows to be transformed were counterbalanced. The third trial paralleled the second except that the red (black) row was stacked to form a cylinder.

In view of the importance the Genevan theory of functions ascribes to the child's understanding of correspondences, children were post-stratified according to three operative levels. Level 1 nonconservers (NCs) were considered to be the more primitive of the two types of nonconservers identified; Level 2 NCs were considered to be the more advanced nonconservers; and conservers were considered to be most advanced developmentally. Children who, at the beginning of the test, failed to place the red and black checkers in a one-to-one correspondence and then failed to respond correctly on all trials were operationally defined as Level 1 NCs. Children who spontaneously placed the red and black checkers in one-to-one correspondence and responded incorrectly on each trial were operationally defined as Level 2 NCs. Children who responded correctly and supplied adequate justification for their responses (e.g., reversibility, addition–subtraction) on at least two of the three trials were operationally defined as conservers. Four children who manifested mastery of one-to-one correspondence and responded correctly on one or more conservation trials without supplying adequate justification were classified as transitional conservers (Piaget, 1941/1965) but were too few in number to be included in the sample.

Results

Scale Construction

For the purpose of data reduction, a priori scales were constructed. The 14 items from the I and S tasks, believed to embody the most elementary correspondences, were aggregated to form Scale 1. The KR-20 reliability (r_{II}) of Scale 1 was .84. The 14 TPO-I and TPO-S items, reflecting same-directional compositions, were aggregated to form Scale 2 ($r_{II} = .85$). Scale 3 consisted of 10 of the 14 items making up the TPR tasks, thought to be the more difficult subarray compositions; four items with zero or negative item-total correlations were not included ($r_{II} = .72$).

Scale 4 consisted of the 7 LD items ($r_{II} = .88$). Children who responded incorrectly in comparing LD arrays *uniformly* indicated that the puppets had the same amount of candy. Scale 5 consisted of 5 of the 7 IS items; two items with negative item-total correlations were not included ($r_{II} = .73$). A scale score is presented as the proportion of items answered correctly. The means and standard deviations of the five scales are presented in Table 1.

The Performance of Preoperational Children

Genevan theory suggests that preoperational children (nonconservers) should succeed on comparison tasks that involve elementary injective and surjective correspondences. To assess the hypothesis that Level 1 and 2 NCs perform better than chance on Scale 1, two t tests for single means were conducted. The expected proportion of correct responses given random responding to the Scale 1 items was .33, because the child had to choose from among three response alternatives. The means of the Level 1 and 2 NCs, reported in Table 1, were significantly greater than the expected mean, $t(14) = 4.02$, $p < .01$, and $t(25) = 11.98$, $p < .001$, respectively.

The theory also suggests that preoperational children should succeed on comparison tasks that involve the composition of subarrays if the subarray comparisons were in the same direction (TPO items making up Scale 2). The means of the Level 1 and 2 NCs were significantly greater than the expected mean, $t(14) = 2.80$, $p < .01$, and $t(25) = 6.10$, $p < .001$, respectively.

In contrast, t tests indicate that the performance of the Level 1 NCs did not differ from expectation on the Scale 3 and 5 items, $t(14) = .90$, n.s., and $t(14) = -1.10$, n.s., respectively. The performance of the Level 2 NCs, however, differed significantly from expectation on Scales 3 and 5, $t(25) = 6.65$, $p < .001$, and $t(25) = 2.28$, $p < .05$, respectively. Given the overwhelming propensity of Level 1 and 2 NCs to respond incorrectly with "same amount" to the LD items, the performance on the Level 1 and 2 NCs on Scale 4 was significantly worse than expected, $t(14) = -8.04$, $p < .001$, and $t(25) = -4.25$, $p < .001$, respectively.

Comparisons by Operative Levels

It was expected that children's performance on each of the scales varies directly with operative level as operationalized by conserver status. One-way analyses of variance, reported in Table 1, were conducted to examine the association between the scale means and conservation performance. Although age is "confounded" with operative level, age was not controlled in the analyses of variance described next. Age is not considered an exogenous variable that affects knowledge of correspondence relations (see Wohlwill, 1973). Age, however, will be treated in the section on factor analysis.

As indicated in Table 1, performance on each scale was significantly related to number conservation. Pairwise comparisons employing Tukey's honest significant difference ($p < .05$) indicated that on Scale 1, Level 2 NCs performed significantly better than Level 1 NCs, and conservers performed significantly better than both Level 1 and 2 NCs. Conservers performed significantly better than Level 1 and 2 NCs on Scales 2, 3, and 4. Conservers and Level 2 NCs performed significantly better than

Table 1
Scale Means, Standard Deviations, and Test Statistics

Scale	Level 1 NCs (15)	Level 2 NCs (26)	Conservers (23)	*F*	*p*
Scale 1 (I + S items)					
M	.61	.80	.94	13.97	.001
SD	.27	.20	.09		
Scale 2 (TPO)					
M	.54	.64	.89	12.31	.001
SD	.29	.26	.10		
Scale 3 (TPR)					
M	.43	.63	.73	7.05	.01
SD	.30	.23	.19		
Scale 4 (LD)					
M	.06	.18	.43	8.33	.001
SD	.13	.30	.34		
Scale 5 (IS)					
M	.25	.50	.65	7.25	.01
SD	.28	.38	.26		

Note. Scores are presented as proportion correct. NC = nonconservers; I = injective; S = surjective; TPO = two-part one-way subarray composition; TPR = two-part reversed subarray composition; LD = length–density.

Level 1 NCs on Scale 5. Tests for trends revealed a highly linear ($p < .001$) relationship between each scale and conserver status. No test for quadratic trends was significant.

The relative difficulty of Scales 1, 2, and 3 and their relation to operative level were examined by means of a multivariate profile analysis (Morrison, 1976). In conducting the profile analysis, two composite variables were created, a priori, from three repeated measures: (a) the difference between Scale 2 and Scale 1, and (b) the difference between Scale 3 and Scale 2. No interaction between scale differences and conserver status was detected, $F(4, 120) = 1.59$, indicating that the size of the scale differences was not conditioned on operative level. The analysis indicated that scale differences were significant, $F(2, 60) = 18.98, p < .001$. Univariate tests collapsing across conserver status indicated that Scale 2 was more difficult than Scale 1, $F(1, 61) = 12.82, p < .001$, and that Scale 3 was more difficult than Scale 2, $F(1, 61) = 14.96, p < .001$.

A second profile analysis comparing Scales 1 and 4 was conducted. No interaction between operative level and scale difference was detected, $F(2, 61) = .74$. The analysis indicated that Scale 4 was significantly more difficult than Scale 1, $F(1, 61) = 199.67, p < .001$. As expected, both analyses indicated, consistent with earlier reported univariate analyses of variance, that scale performance was significantly related to conserver status.

Of the five scales, performance on the Scale 4 (consisting of the LD items) was worst. The mean score obtained by the conservers did not differ significantly from the .33-point estimate obtained under the assumption of random responding. In order to explore further the group differences in performance on Scale 4, a contingency table analysis based on the frequency data presented in Table 2 was conducted.

Because the same-length stimulus pairs were very compelling in eliciting the "same amount" response, subjects were divided into two groups, those who responded incorrectly to every pair of arrays and those who partially overcame that tendency and responded correctly on one or more comparisons. A chi-square test indicated that conserver status was significantly related to responding correctly to at least one of the LD pairs, $\chi^2(2) = 13.36, p < .01$.

Correlations Among the Scales and Factor Analyses

The correlations among the scales, the number conservation test, and age are presented in Table 3. All correlations were significant ($p < .001$). Table 3 also presents the correlation coefficients corrected for attenuation due to unreliability in the scales, but not in the conservation measure or in age. No reliability estimate was obtained for the number conservation test. Age in months, which was derived from records, was assumed to be measured virtually without error. The median correlation among the five scales was .52, and the median corrected correlation was .70. The median correlation between number conservation and the five scales was .45 (corrected, .51), and the median correlation between age and the five scales was .59 (corrected, .64). Two factors bear on the finding that, compared with number conservation, age had slightly higher correlations with the five scales: (a) Age had considerably more variance than the three-valued number-conservation variable, and (b) age was measured with less error than number conservation.

Using the uncorrected correlations, a principal components analysis (see Table 4) was conducted on the five scales and the number conservation test. Because only one factor was extracted, a rotation was not warranted. The analysis was repeated with age added. Again a unitary factor was extracted. The pattern of loadings on the five scales and the number conservation test was highly similar to the pattern of loadings obtained in the first analysis.

Discussion

The results indicate that performance on the comparison tasks is related to number conservation. The relation was linear,

Table 2
Performance on Scale 4 (Length–Density Items)

Proportion correct	Level 1 nonconservers *n*	Level 1 nonconservers %	Level 2 nonconservers *n*	Level 2 nonconservers %	Conservers *n*	Conservers %
.00	12	80	15	58	5	22
.14	1	7	5	19	3	13
.29	1	7	1	4	3	13
.43	1	7	0		2	9
.57	0		3	12	2	9
.71	0		0		4	17
.86	0		0		2	9
1.00	0		2	8	2	9

and operative level did not interact with task difficulty. The Level 1 and 2 NCs tended to perform at better than chance levels on tasks that involved elementary injective and surjective correspondences, as operationalized by Scales 1 and 2. Conservers tended to perform better on all the scales including the scales in which nonconservers tended to manifest a degree of success. These findings are in keeping with the view that children's performance on each of the scales should improve in two ways—at more advanced levels (a) the understanding of more complicated correspondence relations emerges and (b) thinking becomes increasingly arithmetized and exact, allowing performance to continue to improve on elementary tasks of which the young child already manifests a degree of mastery (e.g., Scales 1 and 2).

Rushton, Brainerd, and Pressley (1983) demonstrated that interrelationships among Piagetian constructs tend to be underestimated because of inadequacies in the psychometric properties of the instruments used to operationalize those constructs. In the present study, care was taken to develop reliable measures of the constructs under study. Correlational results reveal considerable consistency across measures. Factor-analytic findings suggest that a unitary, developmental factor underlies performance differences.

The relatively good performance of nonconservers on Scales 1 and 2 suggests a degree of understanding of injective and surjective correspondences. Cowan (1984) partly anticipated these findings. His 5-year-olds were able, without counting, to compare accurately paired arrays in which elements were aligned one-to-one except for an interior gap in one of the two arrays. The configuration he used represents an injective mapping. The Level 1 NCs in the present study were mainly 4-year-olds. In a later study, Cowan (1987b) showed that 5-year-olds' performance on LD-type arrays could be improved by the introduction of guidelines that establish a one-to-one correspondence. The guidelines create an injective mapping between opposing dots (see Cowan, 1987b, p. 150, Figure 2) by making unmatched dots more identifiable.

The relative success of the nonconservers on Scale 2 suggests some capacity to compose subarray relations of the variety greater + greater (or equal) yields greater. The performance of the more advanced, or Level 2, nonconservers on Scale 3 anticipates the development of the concrete-operational child's understanding of countervailing relations and compensation. For example, to make the correct Scale 3 comparison the child must evaluate Bert's advantage in the morning with reference to Ernie's advantage in the afternoon.

All subjects, Level 1 and 2 NCs as well as conservers, had the most difficulty on the Scale 4 comparisons. Scale 4 represents comparisons described by Beilin (1969) and Zimiles (1966) in their static (quasi-)conservation of inequality paradigm. Non-

Table 3
Correlations Among Measures: The Pearson Correlation Coefficients Below the Diagonal and the Corrected Correlation Coefficients Above the Diagonal[a]

Measure	1	2	3	4	5	Number conservation	Age
Scale 1	—	.78	.67	.47	.75	.61	.59
2	.66	—	.88	.53	.76	.57	.64
3	.52	.69	—	.52	.72	.50	.76
4	.40	.46	.41	—	.56	.48	.71
5	.59	.60	.52	.45	—	.51	.66
Number conservation	.56	.53	.42	.45	.43	—	.53
Age	.54	.59	.65	.67	.56	.53	—

[a] $p < .001$ for all Pearson correlation coefficients.

Table 4
Factor Analysis of the Five Scales and Number Conservation Without and With Age

Measure	Loadings	
	Without age	With age
Scale 1	.82	.79
2	.86	.84
3	.78	.78
4	.66	.69
5	.78	.76
Number conservation	.73	.71
Age	—	.84

conserving children, as well as many conservers, responded to the Scale 4 comparisons by indicating that the puppets had the same amount of candy, thus indexing quantity by length. Fuson (1988) and Michie (1984b) also documented the young child's reliance on same-length strategies in comparing these types of arrays. Table 2 indicates that although conservers tended to perform better than nonconservers, the relation between conserver status and performance on Scale 4 was far from perfect. For example, two Level 2 NCs responded correctly to every Scale 4 comparison and five conservers responded incorrectly to every Scale 4 comparison. This type of finding highlights a weakness in Genevan theory with its emphasis on universal features of cognition, namely, the theory's relative lack of concern for within-group differences in cognitive behavior (Schonfeld, 1986). Individual difference variables such as fluid ability (Horn, 1978) and exposure to procedural aids (Michie, 1984a) may have affected scale performance.

Results also bear on the question of continuity in development. On average, the performance advantage of Level 2 NCs in comparison with Level 1 NCs was about the same as the advantage of the conservers in comparison with the Level 2 NCs. The tests for trends and the profile analyses suggest that developmental change in understanding correspondence relations is linear and continuous rather than saltatory (see Kessen, 1962). Beilin (1985) pointed out that "in Piaget's view development is fundamentally *continuous*" (p. 6). The developmental change captured by the relation of the scale scores to number conservation and the extent to which those performances are intertwined with age are consistent with the continuity viewpoint.

An alternative explanation of the findings is that the tasks did not tap the children's understanding of correspondence relations. Rather, the children may have employed an estimation procedure, such as counting (Klahr & Wallace, 1973), which bypasses the deployment of knowledge of array–array correspondence relations; a prior study (Schonfeld, 1986), however, using a different sample *from the same population,* demonstrated the effectiveness of the instructional set in prompting children to inspect the arrays without counting.

Another alternative explanation of the findings is based on perceptual salience. Length, for example, is arguably a salient cue; therefore, performance on Scale 4 items is explained by the salience of the length cue. Such an explanation parallels a view advanced by Wallach (1969) that holds that performance on conservation tasks is based on the criterion stimuli the child employs as indices of quantity. In addition, Wertheimer (cited in Luchins & Luchins, 1970a, 1970b) advanced the view that Gestalt psychology's organizing principles may provide the basis for estimating number. An explanation of children's scale performance based on perceptual phenomena, however, is subject to two related problems.

First, in both Scales 1 and 4, the terminal points of any pair of arrays were aligned. If length were the preoperational child's chief basis for comparing quantity, preoperational children would have performed similarly on Scales 1 and 4. That is, preoperational children would have repeatedly judged the two arrays making up a Scale 1 item to be equal, a result that was not obtained. Children performed much better on Scale 1 than on Scale 4.

Second, an explanation based on perceptual salience is taxed by the problem of determining the source of a cue's salience. More than the issue of the nature of the cue is at stake. To determine whether a cue is salient, one must look to the responding child as well as to the cue. Heidbreder (1933/1961) wrote that a major criticism of Gestalt psychology's organizing principles is that they are unfocused in nature and apply to "varied phenomena" including visual, temporal, and cognitive domains. Furthermore, the organizing principles are ahistorical in conception and not subject to developmental change (Piaget, 1968/1970). The Genevan theory of functions, in contrast, offers a general developmental formulation that provides some basis for generating hypotheses that are relevant to task performance.

References

Beilin, H. (1969). Stimulus and cognitive transformation in conservation. In D. Elkind & J. H. Flavell (Eds.), *Studies in cognitive development* (pp. 409–437). New York: Oxford University Press.

Beilin, H. (1985). Dispensable and core elements in Piaget's research program. *The Genetic Epistemologist, 13,* 1–16.

Brainerd, C. J. (1977). Effects of spatial cues on children's cardinal number judgments. *Developmental Psychology, 13,* 425–430.

Cowan, R. (1984). Children's relative number judgments: One-to-one correspondence, recognition of noncorrespondence, and the influence of cue conflict. *Journal of Experimental Child Psychology, 38,* 515–532.

Cowan, R. (1987a). When do children trust counting as a basis for relative number judgments? *Journal of Experimental Child Psychology, 43,* 328–345.

Cowan, R. (1987b). Assessing children's understanding of one-to-one correspondence. *British Journal of Developmental Psychology, 5,* 149–153.

Davidson, P. (1987). Early function concepts: Their development and relation to certain mathematical and logical abilities. *Child Development, 58,* 1542–1555.

Davidson, P. (1988). Piaget's category-theoretic interpretation of cognitive development: A neglected contribution. *Human Development, 31,* 225–244.

Dean, A. L., & Deist, S. (1980). Children's precocious anticipatory images. *Child Development, 51,* 1040–1049.

Fuson, K. C. (1988). *Children's counting and concepts of number.* New York: Springer-Verlag.

Gibson, J. J. (1941). A critical review of the concept of set in contemporary experimental psychology. *Psychological Bulletin, 38,* 781–817.

Heidbreder, E. (1961). *Seven psychologies.* New York: Appleton-Century-Crofts. (Original work published 1933)

Hooper, F. H., Fitzgerald, J., & Papalia, D. (1971). Piagetian theory and the aging process: Extensions and speculations. *Aging and Human Development, 2,* 3–20.

Horn, J. L. (1978). The nature and development of intellectual abilities. In R. T. Osborne, C. E. Noble, & N. Weyl (Eds.), *Human variation: The biopsychology of age, race, and sex* (pp. 107–136). New York: Academic Press.

Johnson, D. M. (1955). *The psychology of thought and judgment.* New York: Harper.

Kessen, W. (1962). "Stage" and "structure" in the study of the child. *Monographs of the Society for Research in Child Development, 28*(2, Serial No. 83).

Klahr, D., & Wallace, J. G. (1973). The role of quantification operators in the development of conservation of quantity. *Cognitive Psychology, 4,* 301–327.

Luchins, A. S., & Luchins, E. H. (1970a). *Wertheimer's seminars revisited: Problem solving and thinking* (Vol. 1). Albany, NY: Faculty Student Association of the State University of New York at Albany.

Luchins, A. S., & Luchins, E. H. (1970b). *Wertheimer's seminars revisited: Problem solving and thinking* (Vol. 2). Albany, NY: Faculty Student Association of the State University of New York at Albany.

Michie, S. (1984a). Number understanding in preschool children. *British Journal of Educational Psychology, 54,* 245–253.

Michie, S. (1984b). Why preschoolers are reluctant to count spontaneously. *British Journal of Developmental Psychology, 2,* 347–358.

Morrison, D. F. (1976). *Multivariate statistical methods.* New York: McGraw-Hill.

Piaget, J. (1965). *The child's conception of number.* New York: Norton. (Original work published 1941)

Piaget, J. (1970). *Structuralism.* New York: Harper & Row. (Original work published 1968)

Piaget, J. (1971). *Biology and knowledge: An essay on the relations between organic regulations and cognitive processes.* Chicago: University of Chicago Press. (Original work published 1967)

Piaget, J. (1977). Some recent research and its link with a new theory of groupings and conservation based on commutability. *Annals of the New York Academy of Sciences, 291,* 350–358.

Piaget, J., Grize, J.-B., Szeminska, A., & Vinh-Bang. (1977). *Epistemology and psychology of functions.* Dordrecht, Holland: Reidel. (Original work published 1968)

Pufall, P. B., & Shaw, R. E. (1972). Precocious thoughts on number: The long and the short of it. *Child Development, 43,* 62–69.

Pufall, P. B., Shaw, R. E., & Syrdal-Lasky, A. (1973). Development of number conservation: An examination of some predictions from Piaget's stage analysis and equilibration model. *Child Development, 44,* 21–27.

Rushton, J. P., Brainerd, C. J., & Pressley, M. (1983). Behavioral development and construct validity: The principle of aggregation. *Psychological Bulletin, 94,* 18–38.

Saxe, G. B. (1977). A developmental analysis of notational counting. *Child Development, 48,* 1512–1520.

Schonfeld, I. S. (1986). The Genevan and Cattell-Horn conceptions of intelligence compared: Early implementation of numerical solution aids. *Developmental Psychology, 22,* 204–212.

Schwartz, M. M., & Scholnick, E. K. (1970). Scalogram analysis of logical and perceptual components of conservation of discontinuous quantity. *Child Development, 41,* 695–705.

Silverman, I. W., & Rose, A. P. (1982). Compensation and conservation. *Psychological Bulletin, 91,* 80–101.

Wallach, L. (1969). On the bases of conservation. In D. Elkind & J. H. Flavell (Eds.), *Studies in cognitive development* (pp. 191–219). New York: Oxford University Press.

Wohlwill, J. (1973). *The study of behavioral development.* New York: Academic Press.

Woodworth, R. S. (1937). Situation-and-goal set. *American Journal of Psychology, 50,* 130–140.

Zimiles, H. (1966). The development of conservation and differentiation of number. *Monographs of the Society for Research in Child Development, 31* (6, Whole No. 108).

Received May 19, 1988
Revision received January 25, 1989
Accepted April 25, 1989 ■