Introductory Physics
Laboratory Manual
Course 20300

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Introductory Physics

Laboratory Manual

Introduction

The aim of the laboratory exercise is to give the student an insight into the significance of the physical ideas through actual manipulation of apparatus, and to bring him or her into contact with the methods and instruments of physical investigation. Each exercise is designed to teach or reinforce an important law of physics which, in most cases, has already been introduced in the lecture and textbook. Thus the student is expected to be acquainted with the basic ideas and terminology of an experiment before coming to the laboratory.

The exercises in general involve measurements, graphical representation of the data, and calculation of a final result. The student should bear in mind that equipment can malfunction and final results may differ from expected values by what may seem to be large amounts. This does not mean that the exercise is a failure. The success of an experiment lies rather in the degree to which a student has:

- mastered the physical principles involved,
- understood the theory and operation of the instruments used, and
- realized the significance of the final conclusions.

The student should know well in advance which exercise is to be done during a specific laboratory period. The laboratory instructions and the relevant section of the text should be read before coming to the laboratory. All of the apparatus at a laboratory place is entrusted to the care of the student working at that place, and he or she is responsible for it. At the beginning of each laboratory period it is the duty of the student to check over the apparatus and be sure that all of the items listed in the instructions are present and in good condition. Any deficiencies should be reported to the instructor immediately.

The procedure in each of these exercises has been planned so that it is possible for the prepared student to perform the experiment in the scheduled laboratory period. Data sheets should be initialed by your instructor or TA. Each student is required to submit a written report which presents the student's own data, results and the discussion requested in the instructions. Questions that appear in the instructions should be thought about and answered at the corresponding position in the report. Answers should be written as complete sentences.

If possible, reports should be handed in at the end of the laboratory period. However, if this is not possible, they must be submitted no later than the beginning of the next exercise OR the deadline set by your instructor.

Reports will be graded, and when possible, discussed with the student. You may check with the TA about your grade two weeks after you have submitted it.

Laboratory Manners

1. Smoking is not permitted in any college building.
2. Students must not bring food or drinks into the room.
3. Apparatus should not be taken from another position. If something is missing, notify the instructor, and either equipment will be replaced or appropriate adjustments will be made.
4. Students should be distributed as evenly as possible among the available positions. Generally, no more than two students should be working together.
5. At the end of the period the equipment should left neatly arranged for the next class. Non-functioning equipment should be reported before leaving. All papers and personal items have to be removed.

Measurements and Uncertainty

> "A measurement result is complete only when accompanied by a quantitative statement of its uncertainty. The uncertainty is required in order to decide if the result is adequate for its intended purpose and to ascertain if it is consistent with other similar results."
> National Institute of Standards and Technology

1. Introduction

No measuring device can be read to an unlimited number of digits. In addition when we repeat a measurement we often obtain a different value because of changes in conditions that we cannot control. We are therefore uncertain as to the exact values of measurements. These uncertainties make quantities calculated from such measurements uncertain as well.

Finally we will be trying to compare our calculated values with a value from the text in order to verify that the physical principles we are studying are correct. Such comparisons come down to the question "Is the difference between our value and that in the text consistent with the uncertainty in our measurements?".

The topic of measurement involves many ideas. We shall introduce some of them by means of definitions of the corresponding terms and examples.

Sensitivity - The smallest difference that can be read or estimated on a measuring instrument. Generally a fraction of the smallest division appearing on a scale. About 0.5 mm on our rulers. This results in readings being uncertain by at least this much.

Variability - Differences in the value of a measured quantity between repeated measurements. Generally due to uncontrollable changes in conditions such as temperature or initial conditions.

Range - The difference between largest and smallest repeated measurements. Range is a rough measure of variability provided the number of repetitions is large enough. Six repetitions are reasonable. Since range increases with repetitions, we must note the number used.

Uncertainty - How far from the correct value our result might be. Probability theory is needed to make this definition precise, so we use a simplified approach.

We will take the larger of range and sensitivity as our measure of uncertainty.

<ins>Example:</ins> In measuring the width of a piece of paper torn from a book, we might use a cm ruler with a sensitivity of 0.5 mm (0.05 cm), but find upon 6 repetitions that our measurements range from 15.5 cm to 15.9 cm. Our uncertainty would therefore be 0.4 cm.

Precision - How tightly repeated measurements cluster around their average value. The uncertainty described above is really a measure of our precision.

Accuracy - How far the average value might be from the "true" value. A precise value might not be accurate. For example: a stopped clock gives a precise reading, but is rarely accurate. Factors that affect accuracy include how well our instruments are calibrated (the correctness of the marked values) and how well the constants in our calculations are known. Accuracy is affected by systematic errors, that is, mistakes that are repeated with each measurement.

<ins>Example:</ins> Measuring from the end of a ruler where the zero position is 1 mm in from the end.

Blunders - These are actual mistakes, such as reading an instrument pointer on the wrong scale. They often show up when measurements are repeated and differences are larger than the known uncertainty. For example: recording an 8 for a 3, or reading the wrong scale on a meter..

Comparison - In order to confirm the physical principles we are learning, we calculate the value of a constant whose value appears in our text. Since our calculated result has an uncertainty, we will also calculate a Uncertainty Ratio, UR, which is defined as

$$\mathrm{UR} = \frac{|\text{experimental value } - \text{ text value}|}{\text{Uncertainty}}$$

A value less than 1 indicates very good agreement, while values greater than 3 indicate disagreement. Intermediate values need more examination. The uncertainty is not a limit, but a measure of when the measured value begins to be less likely. There is always some chance that the many effects that cause the variability will all affect the measurement in the same way.

<ins>Example:</ins> Do the values 900 and 980 agree?
If the uncertainty is 100 , then $UR = 80/100 = 0.8$ and they agree,
but if the uncertainty is 20 then $UR = 80/20 = 4$ and they do not agree.

2. Combining Measurements

Consider the simple function $R = a\,b$ when a and b have uncertainties of Δa and Δb. Then

$$\Delta R = (a + \Delta a)(b + \Delta b) - a\,b = a\,\Delta b + b\,\Delta a + (\Delta b)(\Delta a)$$

Since uncertainties are generally a few percent of the value of the variables, the last product is much less than the other two terms and can be dropped. Finally, we note that dividing by the original value of R separates the terms by the variables.

$$\frac{\Delta R}{R} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

The RULE for combining uncertainties is given in terms of fractional uncertainties, $\Delta x/x$. It is simply that each factor contributes equally to the fractional uncertainty of the result.

Example: To calculate the acceleration of an object travelling the distance d in time t, we use the relationship: $a = 2\,d\,t^{-2}$. Suppose d and t have uncertainties Δd and Δt, what is the resulting uncertainty in a, Δa?

Note that t is raised to the second power, so that $\Delta t/t$ counts twice. Note also that the numerical factor is the absolute value of the exponent. Being in the denominator counts the same as in the numerator. The result is that

$$\frac{\Delta a}{a} = \frac{\Delta d}{d} + 2\frac{\Delta t}{t}$$

Examination of the individual terms often indicates which measurements contribute the most to the uncertainty of the result. This shows us where more care or a more sensitive measuring instrument is needed.

If $d = 100$ cm, $\Delta d = 1$ cm, $t = 2.4$ s and $\Delta t = 0.2$ s, then $\Delta d/d = (1\text{cm})/(100\text{cm}) = 0.01 = 1\%$ and $2\Delta t/t = 2(0.2\text{s})/(2.4\text{s}) = 0.17 = 17\%$. Clearly the second term controls the uncertainty of the result. Finally, $\Delta a/a = 18\%$. (As you see, fractional uncertainties are most compactly expressed as percentages, and since they are estimates, we round them to one or two meaningful digits.)

Calculating the value of a itself ($2 \times 100/2.4^2$), the calculator will display 34.7222222. However, it is clear that with $\Delta a/a = 18\%$ meaning $\Delta a \approx 6$ cm s^{-2}, most of those digits are meaningless. Our result should be rounded to 35 cm s^{-2} with an uncertainty of 6 cm s^{-2}.

In recording data and calculations we should have a sense of the uncertainty in our values and not write figures that are not significant. Writing an excessive number of digits is incorrect as it indicates an uncertainty only in the last decimal place written.

3. A General Rule for Significant Figures

In multiplication and division we need to count significant figures. These are just the number of digits, starting with the first non-zero digit on the left. For instance: 0.023070 has five significant figures, since we start with the 2 and count the zero in the middle and at the right.

The rule is: Round to the factor or divisor with the fewest significant figures. This can be done either before the multiplication or division, or after.

Example: $7.434 \times 0.26 = 1.93284 = 1.9$ (2 significant figures in 0.26).

4. Reporting Uncertainties

There are two methods for reporting a value V, and its uncertainty U.

A. The technical form is "$(V \pm U)$ units".

Example: A measurement of 7.35 cm with an uncertainty of 0.02 cm would be written as (7.35 ± 0.02) cm. Note the use of parentheses to apply the unit to both parts.

B. Commonly, only the significant figures are reported, without an explicit uncertainty. This implies that the uncertainty is 1 in the last decimal place.

Example: Reporting a result of 7.35 cm implies ± 0.01 cm.

Note that writing 7.352786 cm when the uncertainty is really 0.01 cm is wrong.

C. A special case arises when we have a situation like 1500 ± 100. Scientific notation allows use of a simplified form, reporting the result as 1.5×10^3. In the case of a much smaller uncertainty, 1500 ± 1, we report the result as 1.500×10^3, showing that the zeros on the right are meaningful.

5. Additional Remarks

A. In the technical literature, the uncertainty also called the error.

B. When measured values are in disagreement with standard values, physicists generally look for mistakes (blunders), re-examining their equipment and procedures. Sometimes a single measurement is clearly very different from the others in a set, such as reading the wrong scale on a clock for a single timing. Those values can be ignored, but NOT erased. A note should be written next to any value that is ignored.

Given the limited time we will have, it will not always be possible to find a specific cause for disagreement. However, it is useful to calculate at least a preliminary result while still in the laboratory, so that you have some chance to find mistakes.

C. In adding the absolute values of the fractional uncertainties, we overestimate the total uncertainty since the uncertainties can be either positive or negative. The correct statistical rule is to add the fractional uncertainties in quadrature, i.e.

$$\left(\frac{\Delta y}{y}\right)^2 = \left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2$$

D. The professional method of measuring variation is to use the Standard-Deviation of many repeated measurements. This is the square root of the total squared deviations from the mean, divided by the square root of the number of repetitions. It is also called the Root-Mean-Square error.

E. Measurements and the quantities calculated from them usually have units. Where values are tabulated, the units may be written once as part of the label for that column The units used must appear in order to avoid confusion. There is a big difference between 15 mm, 15 cm and 15 m.

Graphical Representation of Data

Graphs are an important technique for presenting scientific data. Graphs can be used to suggest physical relationships, compare relationships with data, and determine parameters such as the slope of a straight line.

There is a specific sequence of steps to follow in preparing a graph. (See Figure 1)

1. Arrange the data to be plotted in a table.

2. Decide which quantity is to be plotted on the x-axis (the abscissa), usually the independent variable, and which on the y-axis (the ordinate), usually the dependent variable.

3. Decide whether or not the origin is to appear on the graph. Some uses of graphs require the origin to appear, even though it is not actually part of the data, for example, if an intercept is to be determined.

4. Choose a scale for each axis, that is, how many units on each axis represent a convenient number of the units of the variable represented on that axis. (Example: 5 divisions = 25 cm)

   Scales should be chosen so that the data span almost all of the graph paper, and also make it easy to locate arbitrary quantities on the graph. (Example: 5 divisions = 23 cm is a poor choice.) Label the major divisions on each axis.

5. Write a label in the margin next to each axis which indicates the quantity being represented and its units.

   Write a label in the margin at the top of the graph that indicates the nature of the graph, and the date the data were collected. (Example: "Air track: Acceleration vs. Number of blocks, 12/13/05")

6. Plot each point. The recommended style is a dot surrounded by a small circle. A small cross or plus sign may also be used.

7. Draw a smooth curve that comes reasonably close to all of the points. Whenever possible we plot the data or simple functions of the data so that a straight line is expected.

   A transparent ruler or the edge of a clear plastic sheet can be used to "eyeball" a reasonable fitting straight line, with equal numbers of points on each side of the line. Draw a single line all the way across the page. Do not simply connect the dots.

8. If the slope of the line is to be determined, choose two points on the line whose values are easily read and that span almost the full width of the graph. These points should not be original data points.

   Remember that the slope has units that are the ratio of the units on the two axes.

9. The uncertainty of the slope may be estimated as the larger uncertainty of the two end points, divided by the interval between them.

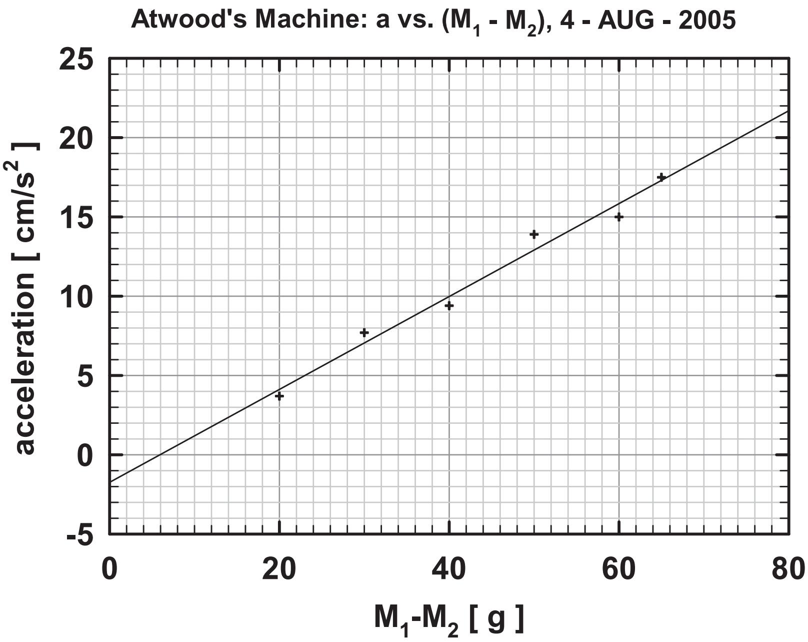


Figure 1: Example graph.

Using Figure 1 as an example, the slope of the straight line shown may be calculated from the values at the left and right edges, (-1.8 cm/s^{-}2 at 0 g and 21.8 cm/s^2 at 80 g) to give the value:

$$\text{Slope} = \frac{(21.8 - (-1.8))\ \text{cm/s}^2}{(80 - 0)\ \text{g}} = \frac{23.6\ \text{cm/s}^2}{80\ \text{g}} = 0.295 \frac{\text{cm}}{\text{s}^2\ \text{g}}$$

Suppose that the uncertainty is about 1.0 cm/s^2 at the 70 g value. The uncertainty in the slope would then be (1.0 cm/s^2)/(70 - 20) g = 0.02 cm/(s^2 g). We should then report the slope as (0.30 ± 0.02) cm/(s^2 g). (Note the rounding to 2 significant figures.)

If the value of g (the acceleration of free fall) in this experiment is supposed to equal the slope times 3200 g, then our experimental result is

$$3200\ \text{g} \times (0.30 \pm 0.02)\ \text{cm/(s}^2\ \text{g)} = (9.60 \pm 0.64)\ \text{m/s}^2$$

To compare with the standard value of 9.81 m/s^2, we calculate the uncertainty ratio, UR.

$$\text{UR} = (9.81 - 9.60)/0.64 = 0.21/0.64 = 0.33,$$

so the agreement is very good.

[Note: Making the uncertainty too large (lower precision) can make the result appear in better agreement (seem more accurate), but makes the measurement less meaningful.]

The Vernier Caliper

A vernier is a device that extends the sensitivity of a scale. It consists of a parallel scale whose divisions are less than that of the main scale by a small fraction, typically 1/10 of a division. Each vernier division is then 9/10 of the divisions on the main scale. The lower scale in Fig. 2 is the vernier scale, the upper one, extending to 120 mm is the main scale.

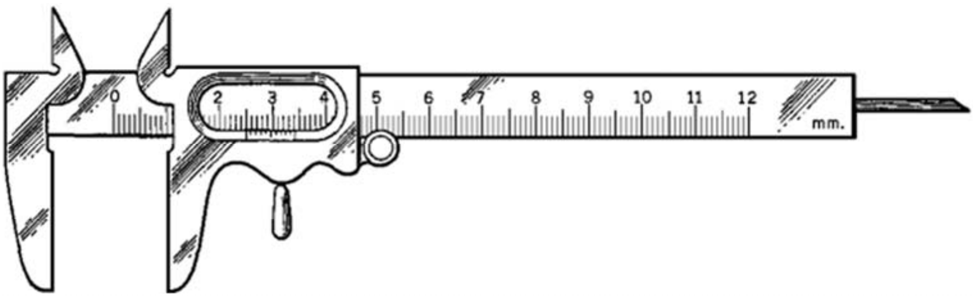
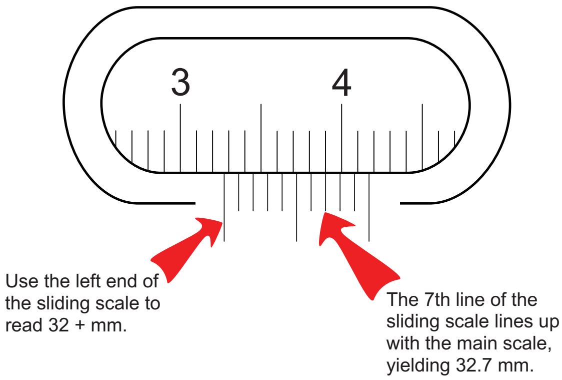


Figure 2: Vernier Caliper.

The left edge of the vernier is called the index, or pointer. The position of the index is what is to be read. When the index is beyond a line on the main scale by 1/10 then the first vernier line after the index will line up with the next main scale line. If the index is beyond by 2/10 then the second vernier line will line up with the second main scale line, and so forth.



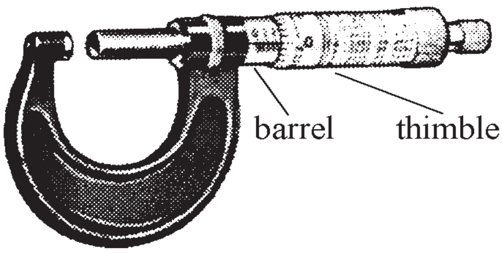
If you line up the index with the zero position on the main scale you will see that the ten divisions on the vernier span only nine divisions on the main scale. (It is always a good idea to check that the vernier index lines up with zero when the caliper is completely closed. Otherwise this zero reading might have to be subtracted from all measurements.)

Note how the vernier lines on either side of the matching line are inside those of the main scale. This pattern can help you locate the matching line.

The sensitivity of the vernier caliper is then 1/10 that of the main scale. Keep in mind that the variability of the object being measured may be much larger than this. Also be aware that too much pressure on the caliper slide may distort the object being measured.

The Micrometer Caliper

Also called a screw micrometer, this measuring device consists of a screw of pitch 0.5 mm and two scales, as shown in Fig. 3. A linear scale along the barrel is divided into half millimeters, and the other is along the curved edge of the thimble, with 50 divisions.



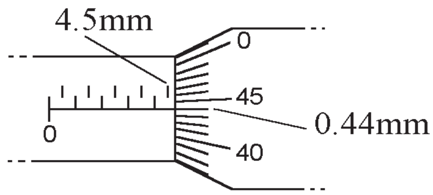


Figure 3: Micrometer Caliper.

The pointer for the linear scale is the edge of the thimble, while that for the curved scale is the solid line on the linear scale. The reading is the sum of the two parts in mm . The divisions on the linear scale are equal to the pitch, 0.5 mm. Since this corresponds to one revolution of the thimble, with its 50 divisions, then each division on the thimble corresponds to a linear shift of $(0.50 \text{ mm})/50 = 0.01 \text{ mm}$.

In Fig. 3, the value on the linear scale can be read as 4.5 mm , and the thimble reading is $44 \times 0.01 \text{ mm} = 0.44 \text{ mm}$. The reading of the micrometer is then $(4.50 + 0.44) \text{ mm} = 4.94 \text{ mm}$.

Since a screw of this pitch can exert a considerable force on an object between the spindle and anvil, we use a ratchet at the end of the spindle to limit the force applied and thereby, the distortion of the object being measured. The micrometer zero reading should be checked by using the ratchet to close the spindle directly on the anvil. If it is not zero, then this value will have to be subtracted from all other readings.

Angle Scale Verniers

This type of vernier appears on spectrometers, where a precise measure of angle is required. Angles arc measured in degrees (°) and minutes ('), where 1 degree = 60 minutes. Fig. 4 shows an enlarged view of a typical spectrometer vernier, against a main scale which is divided in 0.5° = 30'.

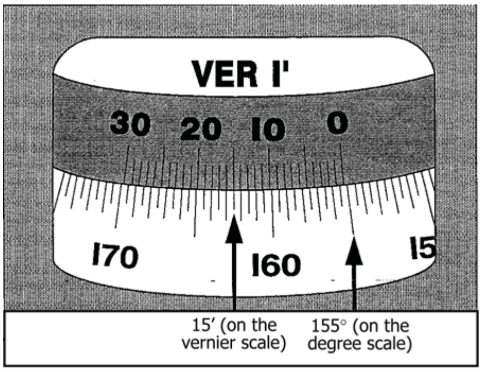


Figure 4: Angle Scale Vernier.

The Vernier has 30 divisions, so that the sensitivity of the vernier is one minute. (There are also two extra divisions, one before 0 and the other after 30, to assist in checking for those values.) Each division on the vernier is by 1/30 smaller than the division of the main scale. When the index is beyond a main scale line by 1/30 of a division or 1', line 1 on the vernier is lined up with the next main scale line. When that difference is 2/30 or 2', line 2 on the vernier lines up with the next line on the main scale, and so on.

Fig. 4 shows an example where degree and Vernier scale run from right to left. Again, reading the angle is a two step process. First we note the position of the index (zero line on the Vernier) on the main scale. In the figure it is just beyond 155.0°. To read the vernier, we note that line 15 seems to be the best match between a vernier line and a main scale line. The reading is then $155.0° + 15' = 155° \, 15' = 155.25°$.

The example shows one problem with working with angles, the common necessity of converting between decimal fraction and degree-minute-second (DMS) notation. We illustrate another place where this arises with the problem of determining the angle between the direction of light entering the spectrometer, and the telescope used to observe light of a particular wavelength.

Example: The position of the telescope to observe the zeroth diffraction order is $121°55'$. Light of a certain wavelength is observed at $138°48'$. The steps in the subtraction are illustrated below, using DMS and decimal notation, respectively. Either method is correct.

	DMS	decimal
$138° \, 48'$	$137° \, 108'$	$138.80°$
$-\ 121° \, 55'$	$-\ 121° \quad 55'$	$-\ 121.92°$
?????	$16° \quad 53'$	$16.88°$

Vectors - Equilibrium of a Particle

APPARATUS

1. A force table equipped with a ring, pin, four pulleys, cords and pans
2. A set of 16 slotted masses: Set of known masses (slotted type)
   (4 × 100 g, 4 × 50 g, 2 × 20 g, 2 × 10 g, 1 × 5 g, 2 × 2 g, 1 × 1 g)
3. Protractor
4. Ruler

INTRODUCTION

Physical quantities that require both a magnitude and direction for their description are vector quantities. Vectors must be added by special rules that take both parts of the description into account. One method for adding vectors is graphical, constructing a diagram in which the vectors are represented by arrows drawn to scale and oriented with respect to a fixed direction.

Using the graphic method we can rapidly solve problems involving the equilibrium of a particle, in which the vector sum of the forces acting on the particle must be equal to zero. While the graphical method has lower accuracy than analytical methods, it is a way of getting a feel for the relative magnitude and direction of the forces. It can also solve the problem of the ambiguity of the direction of a force where the analytical method uses an arctangent to determine direction, which gives angles in the range $-90°$ to $+90°$.

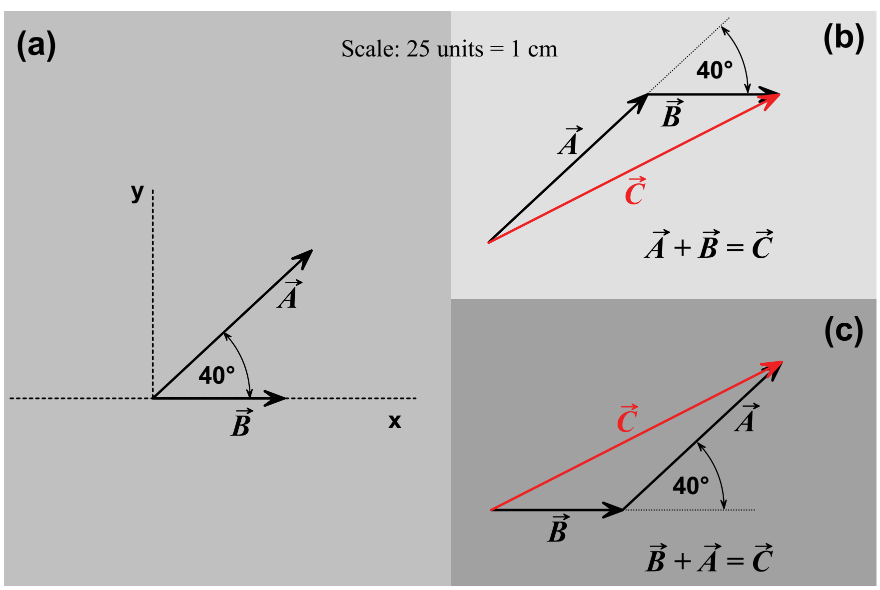


Figure 1: Graphical addition of two vectors

The graphical addition of two vectors is illustrated in Figure 1. A suitable scale must be chosen, so that the diagram will be large enough to fill most of the work space. The scale is written in the work space in the form of an equation, as appears at the top of Figure 1. This is read as ”25 units of force are represented by 1 cm on the page”.

A 0° direction should also be indicated, for example the x axis in Figure 1(a). Figure 1(b) illustrates starting with $\vec{A}$, while Figure 1(c) starts with $\vec{B}$. In each case the second vector is drawn from the head of the first vector (this is the head-to-tail rule). Note that the direction of the second vector is measured from the direction of the first vector as shown in Figure 1(c).

Note also that the vectors are labeled as they are completed.

You can think of this process as "walking" along segments of a path. The equivalent walk, or **resultant vector**, goes from the beginning (tail) of the first vector to the head of the last vector. This is the vector $\vec{C}$ shown in Figures 1(b) and 1(c). The length of this vector can be measured and the scale used to convert back to force units. The direction can also be measured, so that the force vector is then completely described.

In the addition of more than two vectors the procedure is the same, as shown in Figure 2. Vectors are drawn head-to-tail. Shown is $\vec{B} + \vec{A} + \vec{D} = \vec{E}$.

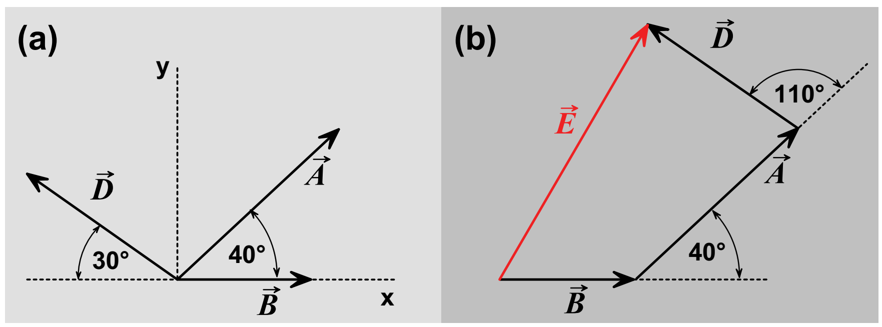


Figure 2: Graphical addition of three vectors

NOTES AND TECHNIQUES

1. We will use "grams" as a unit of force. While not strictly correct, the weight of a given mass is proportional to that mass.

2. The central ring has spokes that connect the inner and outer edges. When connecting the cords be sure that the hooks have not snagged on a spoke. In that case the cord will not be radial. Also whenever you change a pulley position, check that the cord is still radial.

3. Be sure that when you position a pulley, that both edges of the clamp arc snugly against the edge of the force table. Check that the cord is on the pulley.

4. There are two tests for equilibrium.
   The first test is to move the pin up and down and observe the ring. If it moves with the pin, the system is NOT in equilibrium and forces need to be adjusted.
   The second test is to remove the pin. However, this should be done in two stages. First just lift it but hold it in the ring to prevent large motions. If there is no motion, remove it completely. If the ring remains centered then the system is in equilibrium.

5. The circle diagram is a quick sketch of the top of the table and summarizes the forces acting on the ring (the particle). Do not spend time making this "pretty". The vector addition diagrams are to be done carefully with ruler and protractor.

PROCEDURE

Part I: Resultant and Equilibrant of Two Forces

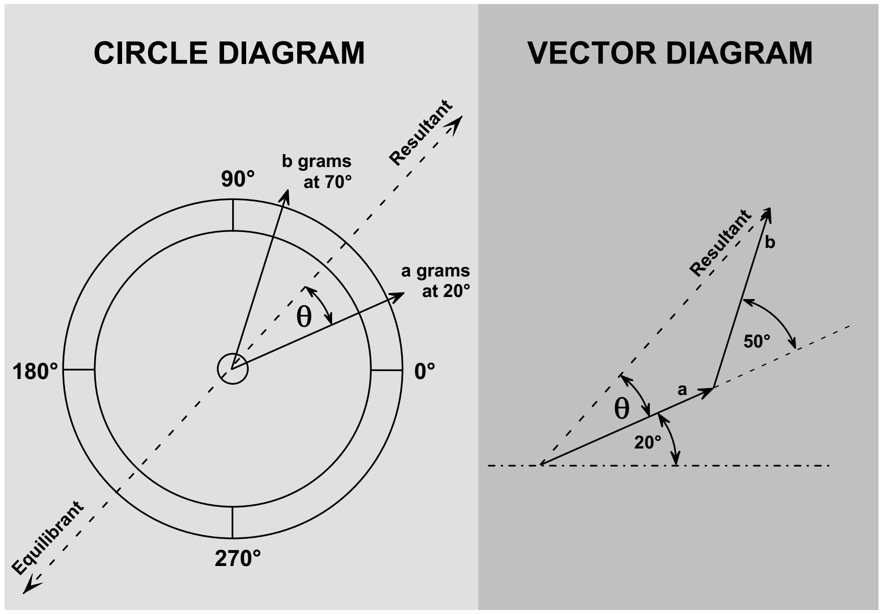


Figure 3

(a) Mount one pulley on the force table at the 0° mark. Mount a second pulley at any angle between 50° and 80°. Suspend unequal loads on cords running from the central disk over the pulleys. Record the angles and the loads on a circle diagram. Remember that the pans weigh 50 grams.

(b) Choose a scale so that the sum of the two forces would almost reach across the page. Draw a scale diagram of the two vectors in the "head to tail" position, as in the $\vec{A} - \vec{B}$ portion of Figures 1(c) or 2(b). Complete the diagram so as to determine the magnitude and direction of the resultant, as in Figure 1(c). The resultant is the single force that is the sum of the two original forces.
Write down the size of the resultant and its direction.

(c) The equilibrant is the single force that would balance the original two forces. It should be equal in size to the resultant, and opposite in direction. Mount a pulley at this angle on the force table, and use a cord to apply the appropriate load. Perform the first test for equilibrium (moving the pin).
You may have to vary the load slightly, or the angle to obtain a good balance. When you think this has been achieved, perform the second test (removing the pin). Record the final values of the load and angle.

Questions (to be answered in your report): How close did the final values come to the values determined from the vector addition diagram? (You should have been able to get within 3 grams and 1.5 degrees.)

Part II: Addition of Four Forces Producing Equilibrium

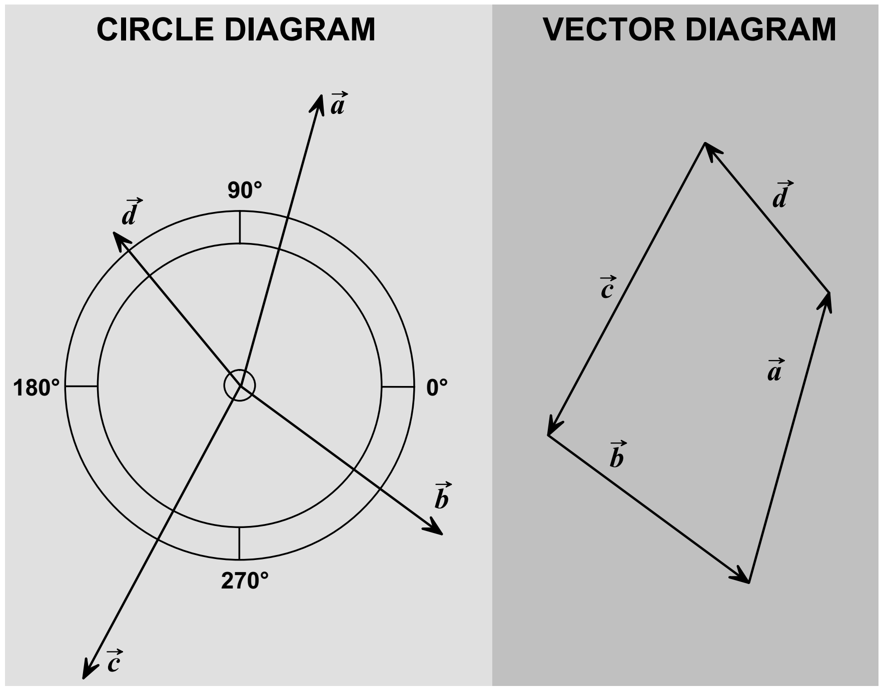


Figure 4

(a) Set up four pulleys and suspend unequal loads on the cords running over them. Change angles and loads until the system is in equilibrium (i.e. passes both tests). Be sure that the cords are still radial. Sketch the circle diagram to record loads (remember the weight of the pans) and angles.

(b) Carefully draw the vector addition diagram (see figure 4). Note that your diagram may not close due to small errors.

Questions (to be answered in your report):

1. How large is the resultant of the four force vectors in your diagram?

2. Why should we expect the vector addition diagram to close?

(c) The discrepancy can arise from two sources, errors in the lengths of the lines in our diagram and errors in angles. We can estimate the uncertainty in each by noting the sensitivity of the ruler (ΔL) and protractor ($\Delta\theta$). (Sensitivity is the smallest quantity that can be read or estimated from a scale.) Convert ΔL to a force by using your scale value. Convert $\Delta\theta$ to radians and multiply by twice the largest force in the diagram. ($180° = \pi$ Rad.) The sum of these two terms is an estimate of the uncertainty in the resultant.

Questions (to be answered in your report): How does the size of your resultant in (b) compare with this uncertainty?

Part III: Determination of X and Y Components of a Force

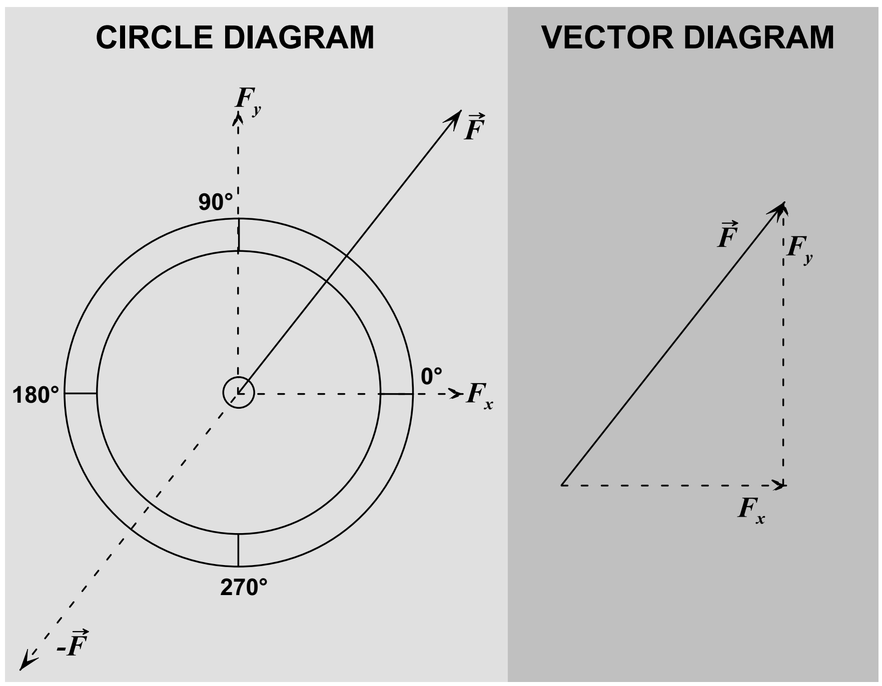


Figure 5

(a) Place a pulley on the 30° mark of the force table and apply a load over it. Note the total load and angle at the top of a data sheet.

(b) If the resultant of the vector sum of two forces is the single force that is an exact equivalent of the two original forces, then we can reverse the process and find the two forces, in convenient directions, that is equivalent to any given single force.
Draw a set of X-Y axes at the lower left of your data sheet. Choose a scale so that the vector representing your load will span most of the page. Draw the vector representing your load, assuming that the positive X axis is the 0° direction.
Drop a perpendicular to the X axis from the head of your vector (this line makes an angle of 60° with the direction of the vector).
Draw arrow heads on the two legs of the resulting right triangle, as if they were two vectors that added to the vector on the hypotenuse.

(c) The vector along the X axis and the vector parallel to the Y axis are called component vectors[1]. Convert to force values and sketch a circle diagram with the results.
Set up these forces on your force table. Move the original load by 180°, to 210°.
Test for equilibrium, and make any necessary adjustments to the load to balance.

Questions (to be answered in your report): : How large an adjustment did you have to make? How does this compare with the uncertainty you found in II.(c)?

(d) The sides of the right triangle can also be determined by trigonometry. Calculate the size of the component vectors trigonometrically. Show your calculation. How do they compare with the values you found graphically ?

[1]Components are scalars that have an accompanying indication of direction.

Air Track

APPARATUS

1. Air Track with air blower
2. Glider with spring bumpers
3. Stop clock
4. Set of four 1.27 cm thick spacers
5. Meter stick (and vernier caliper)

INTRODUCTION

The air track is a long hollow aluminum casting with many tiny holes in the surface. Air blown out of these holes provides an almost frictionless cushion of air on which the glider can move. The air track and gliders operate best if they are clean and smooth. If their surfaces are dirty or show bumps or nicks, inform your instructor before proceeding. Dirt, bumps, and nicks can result in scratching the surfaces of the track and glider. To avoid scratching, use care in handling the apparatus. The most important rule is this: *at no time should the glider be placed on the air track if the blower is not in operation.*

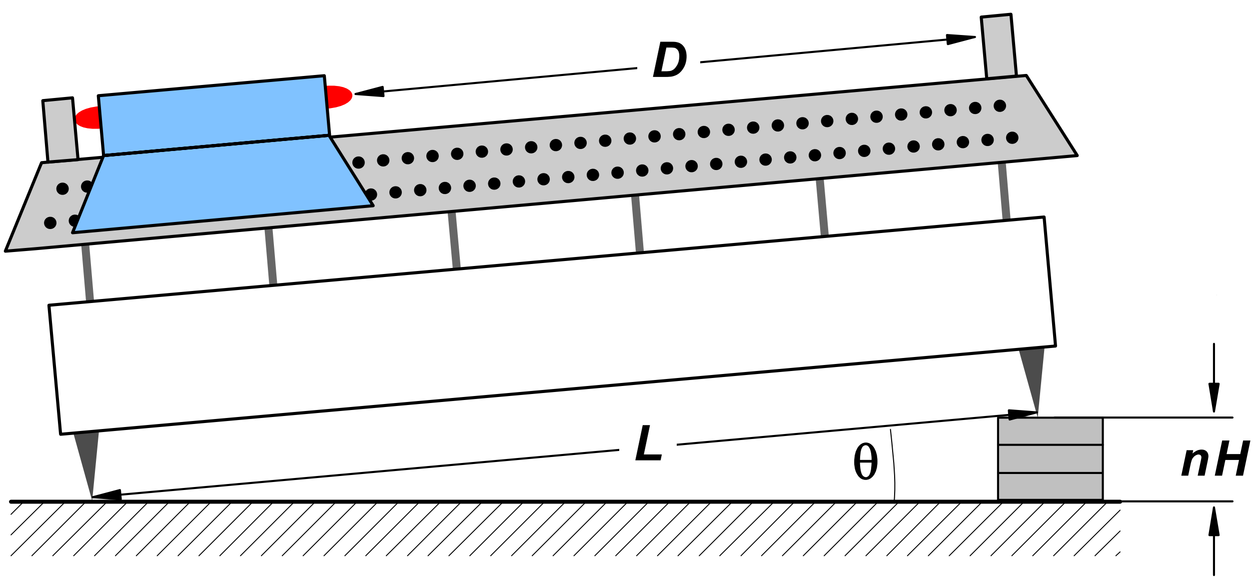


Figure 1: Air Track with dimensions.

The air track, with air blowing, serves as an almost frictionless surface. When one end is raised, it becomes a frictionless inclined plane (see Fig. 1). According to theory, the acceleration a of an object due to gravity down a frictionless incline of angle θ is $a = g\ sin\,\theta$. The main idea of this experiment is two-fold: first, to check that the acceleration is proportional to $sin\,\theta$, and second, to find the value of g, the acceleration of free fall.

The acceleration down the track is to be found for each of four incline angles by measuring the time t required for the glider to slide the measured distance D down the length of the track, and, using the formula $D = \frac{1}{2}\,a\,t^2$, to get $a = 2\,D/t^2$ (see Fig. 1).

The collision between the glider and the spring bumper at the bottom of the track is not perfectly elastic, that is, the glider rebounds at lower speed and less kinetic energy than it had just before the collision. This is demonstrated by the fact that the glider rebounds a distance $D' < D$. The potential energy of the glider with respect to the bottom of the track just before release is $m\ g\ D\ sin\,\theta$, while the potential energy at standstill after rebound is $m\ g\ D'\ sin\,\theta$. These are also the kinetic energies just immediately before (K) and immediately after (K') the collision, respectively. The fractional loss of energy is then: $(K - K')/K = 1 - (K'/K) = 1 - (D'/D)$.

PROCEDURE

Part A: Determining the acceleration

1. Check that the air track is level (horizontal). It should take the glider at least 10 seconds to move across the track, no matter at which end it is placed. If it is not sufficiently level check with your instructor.

2. Measure:

   a) The distance D the glider moves in going from one end of the track to the other.
   b) The distance L between the base supports of the air track.

3. Place one spacer under the single track support. With the air on, place the glider at the upper end with about one millimeter gap between the glider and the stationary spring. Take a few practice timings to get used to starting the clock at the time the glider is released, and stopping the clock when it hits the spring at the lower end.
   Be careful when reading the clock. The two dials have different divisions.
   When you are used to the process of release and timing, then proceed.

4. Set up a table with the headings shown below:

n (# of blocks)	t_1 (s)	t_2 (s)	$t_{average}(s)$	a (m/s^2)
1				
2				

   Be sure to indicate the units used. For each value of n, from 1 to 4, measure the time twice. If they differ by more than 5%, then take another pair of times. Continue until two successive times are within 5%.

5. With four blocks in place, $n = 4$, take a total of six times. We will use this information to determine the variability in our time measurements.

Part B: Determining the dissipated energy

1. With two spacers, measure the rebound distance D'. Use the average of two successive measurements that are within 5% of each other.

2. Evaluate the fraction of the original kinetic energy that is lost in the collision.

3. Explain the equations relating K and K' to D and D' and then derive $K'/K = D'/D$.

ANALYSIS

1. If friction has been eliminated, what are the forces exerted on the glider?
   Draw the Free Body Diagram of the glider.
   Find the net force and apply Newton's Second Law to determine the algebraic relation between the acceleration a, g and the angle θ of the track.
   Use the fact that $sin\,\theta = n\ H/L$ to express a in terms of n algebraically.
   What kind of a graph would you expect for a vs. n?

2. Review the material in the introduction to the lab manual on graphing, and using graphs. Select scales so that the graph of a vs. n takes up most of the page. Do include the origin. Plot your data points. Draw the single straight line that best fits your data points. Determine the slope of that line by using two widely separated points on the line that are not data points. What are the units of the slope?

3. Use the analysis of step 2 to relate your value of the slope to g. Determine g from your slope.

4. In order to compare your value of g with the standard value, a value for the experimental uncertainty is needed. Review the material in the introduction on "Measurements and Uncertainty" and on using graphs. Determine the range of the six times measured for $n = 4$. Take this value as the uncertainty in time measurements, Δt. Determine the uncertainty in the value of acceleration Δa due to Δt. The uncertainly in the slope measurement is just Δa divided by the range of n. Finally, determine Δg from the relation between g and the slope.

5. Compare your value with the standard value by calculating the Uncertainty Ratio:

$$\frac{|g - g_{standard}|}{\Delta g}$$

   Values less than 1 indicate excellent agreement, greater than 4, disagreement and possible mistakes. Values between 1 and 4 are ambiguous, indicating fair or poor agreement. How well does your result agree with the standard value?

Air Track

1. Draw a free-body diagram showing the forces on the glider sitting on an airtrack at an angle θ.

Derive the expression for a in terms of g and θ.

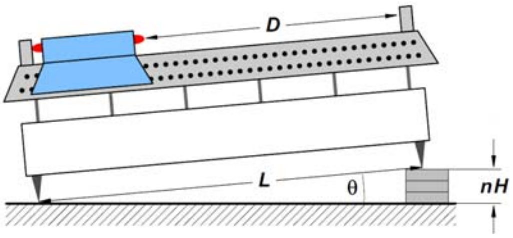
Note $\sin\theta = nH/L$, then a = ____________________ (in terms of g, L, nH). (Where n are the number of blocks and H is their height.

Sketch a graph showing how a will depend on n.

2. Check that the air track is level (horizontal). It should take the glider at least 10 seconds to move across the track, no matter at which end it is placed. If it is not sufficiently level, check with your instructor.

3. Measure D and L for the track

D=______________



L=______________

H=1.27cm

Do not forget units.

3. Releasing the glider and timing should be done by one person. Place one spacer under the single track support (n=1). With the air on, place the glider at the upper end with about one millimeter gap between the glider and the stationary spring. Take a few practice timings to get used to starting the clock at the time the glider is released, and stopping the clock when it hits the spring at the lower end. Be careful when reading the clock. The two dials have different divisions. When you are used to the process of release and timing, proceed.

4. What is the expression of acceleration a in terms of D and t?

a=___________________

5. Measure t and calculate a. With four blocks (n=4), take a total of 6 times.

For n=3, first predict the time, **then get the TA or professor to initial your prediction;** then measure it. If your prediction is not close recalculate and do it again.

n	t_1(s)	t_2(s)	$t_{average}$(s)	a(m/s^2)
1				
2				
4				
3 - predict				
3				

6. Input and plot date points (n,a) on the computer. Graph linear fit on the computer. Determine the slope and its unit.

Slope = _______________(   )

The expression of your slope = ________________

That gives you, g = _______________

Put your value of g on the blackboard. What is the range of values of g found in the class?

How does your g compare with 9.8 m/s^2? Why do you think your value is bigger/smaller than the textbook value?

7. Determine the range of the six times measured for n = 4.Take this value as the uncertainty in time measurements:

In terms of t_1, t_2, t_3, and t_4, write down the expression of Δt =____________________

Then calculate the value of Δt.

Determine the uncertainty in the value of acceleration Δa due to Δt:
First, write down the relationship between a and t:

Then according to the formula you just wrote, derive the expression of Δa due to Δt, then calculate its value.

Finally, determine Δg:

*8. K is the kinetic energy of the glider before the collision and K' is the kinetic energy of the glider after the collision. D is the distance the glider travels all the way from top to bottom and D' is the distance the glider can rebound for. Please deduct that fractional loss of energy during the collision (K-K')/K=1-(K/K')=1-(D/D').

Atwood's Machine

APPARATUS

1. The apparatus consists of two composite masses connected by a flexible wire that runs over two ball-bearing pulleys. The make up of the composite masses at the beginning of the experiment is:

	Left Side	Right Side
	1×1 g	
	2×2 g	
	4×5 g	
	1×10 g	
	1×250 g	1×250 g
	1×500 g	1×500 g
	1×965 g	1×1000 g
Total mass:	1750 g	1750 g

2. Stop clock
3. Pair of tweezers
4. Ruler

INTRODUCTION

This Atwood's machine consists essentially of a wire passing over a pulley with a cylindrical mass attached to each end of the string, The cylinders are composed of three sections, the lower ones of 250 g, and the middle ones of 500 g. Note that the two top-most sections (the sections to which the wire is tied) , do not have the same mass. The one on the left has a mass of 965 g. The right hand one has a mass of 1000 g. Each of these sections has eight vertical holes drilled into its top. When all the small masses are in the left cylinder, the two cylinders have the same mass and the force $(M_1 - M_2)g = 0$. In other words, there is no unbalanced force and the system remains at rest when the brake is released the 10 g mass is transferred from the left to the right hand cylinder, the difference between the two masses becomes $(M_1 - M_2) = 20$ g, while the sum $(M_1 + M_2)$ remains unchanged. If now an additional 5 g mass is transferred, the mass difference becomes 30 g, while the sum is still unchanged, etc. With the small masses provided, it is possible to vary $(M_1 - M_2)$ in 2 g steps from 0 to 70 g.

A string, whose mass per unit length is approximately the as that of the wire, hangs from the two masses. It serves to keep the mass of the string plus wire on each side approximately constant as the system moves, therefore it keeps the accelerating force constant.

PRECAUTIONS

In order to obtain satisfactory results in this experiment and in order to prevent damage to the apparatus, it is necessary to observe the following precautions: Release the brake only when the difference in mass on the two sides is less than 70 g. Preparatory to taking a run, raise the right hand mass until it just touches the bumper. Make sure that it does not raise the movable plate. Always release the brake when moving the masses. Keep your feet away from the descending mass. Before releasing the brake, make sure that the left hand mass is not swinging.

Always stand clear of the suspended masses. The wire may break.

PROCEDURE

Start with a total load of 3500 g and all of the small masses on the left side. Move the left mass (M_2) to its lowest position, (see Fig. 1) Measure the displacement s, which is the distance from the top of the left mass to the bumper.

Transfer 10 g from the left to the right side (i.e. from M_2 to M_1, remember: if 10 g are transferred, then $M_1 - M_2$ will be 20 g). Make two determinations of the time of rise of the left hand mass through the measured distance. If the two time determinations differ by more than 5%, repeat the measurement until you obtain agreement within 5%. Compute the average acceleration using the average of the values of the time. It will be well to practice the timing before recording any results.

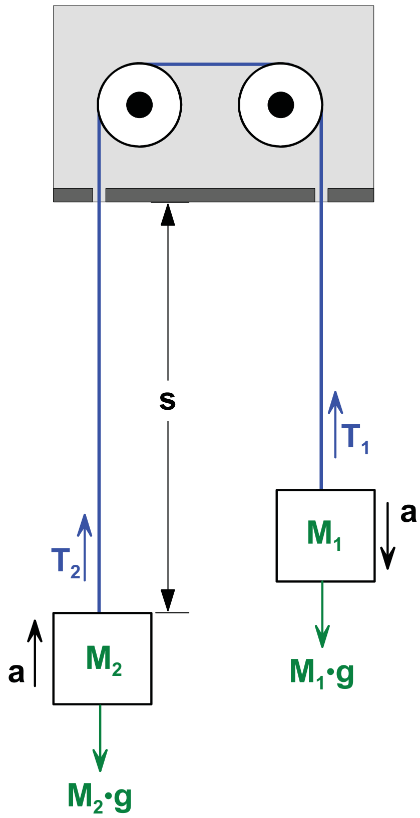


Figure 1: Atwood's machine

Increase the mass difference ($M_1 - M_2$) by about 10 g noting the time required in each case and compute the corresponding accelerations. At least six different mass differences should be used. Setup the data table in the following way, do not forget to note $M_1 + M_2$:

$M_1 - M_2$	time of rise [s]			displacement	acceleration
[g]	first	second	average	s [m]	$a = 2s/t^2$ [m/s^2]

ANALYSIS

Applying Newton's second law to the descending mass (see Fig. 1) we have,

$$M_1\, g - T_1 = M_1\, a \tag{1}$$

and to the ascending mass,

$$T_2 - M_2\, g = M_2\, a \tag{2}$$

where T_1 is the tension in the wire above the descending mass, T_2 the tension in the wire above the ascending mass, and g the acceleration due to gravity. T_1 will be greater than T_2 because there is friction and also because the wheels over which the wire runs are not without some mass, that means, a torque is required to accelerate them.

Adding (1) and (2) and solving for the acceleration we get

$$a = \frac{g}{M_1 + M_2}(M_1 - M_2) - \frac{T_1 - T_2}{M_1 + M_2} \tag{3}$$

where $(M_1 + M_2)$ is constant and $(T_1 - T_2)$ may also be considered as a constant if we assume that the friction remains constant as long as the total mass of the system does not change. If we plot the acceleration a versus the mass differences $(M_1 - M_2)$, then equation (3) is represented by a straight line of slope $g/(M_1 + M_2)$ and intercept $(T_1 - T_2)/(M_1 + M_2)$.

Consult the introduction to this manual for instructions concerning the graphing of data. Plot mass differences on the x-axis and corresponding accelerations the y-axis. Plot the data obtained on graph paper and draw the regression line which best "fits" the points. From measurements of slope and intercept, calculate g and $(T_1 - T_2)$. Your report should show your data table, graph, the method used to determine the slope and your calculations.

Questions (to be answered in your report):

1. What is the advantage of transferring mass from one side to the other, instead of adding mass to one side?
2. How would your results be changed if you gave the system an initial velocity other than zero?
3. Solve for the tension in the wire above the descending mass for the case of the largest acceleration. What would be the tension if $a = g$?

Centripetal Force

APPARATUS

1. Centripetal force apparatus
2. Set of slotted weights
3. Equal-arm balance with standard weights
4. Electric stop-clock

INTRODUCTION

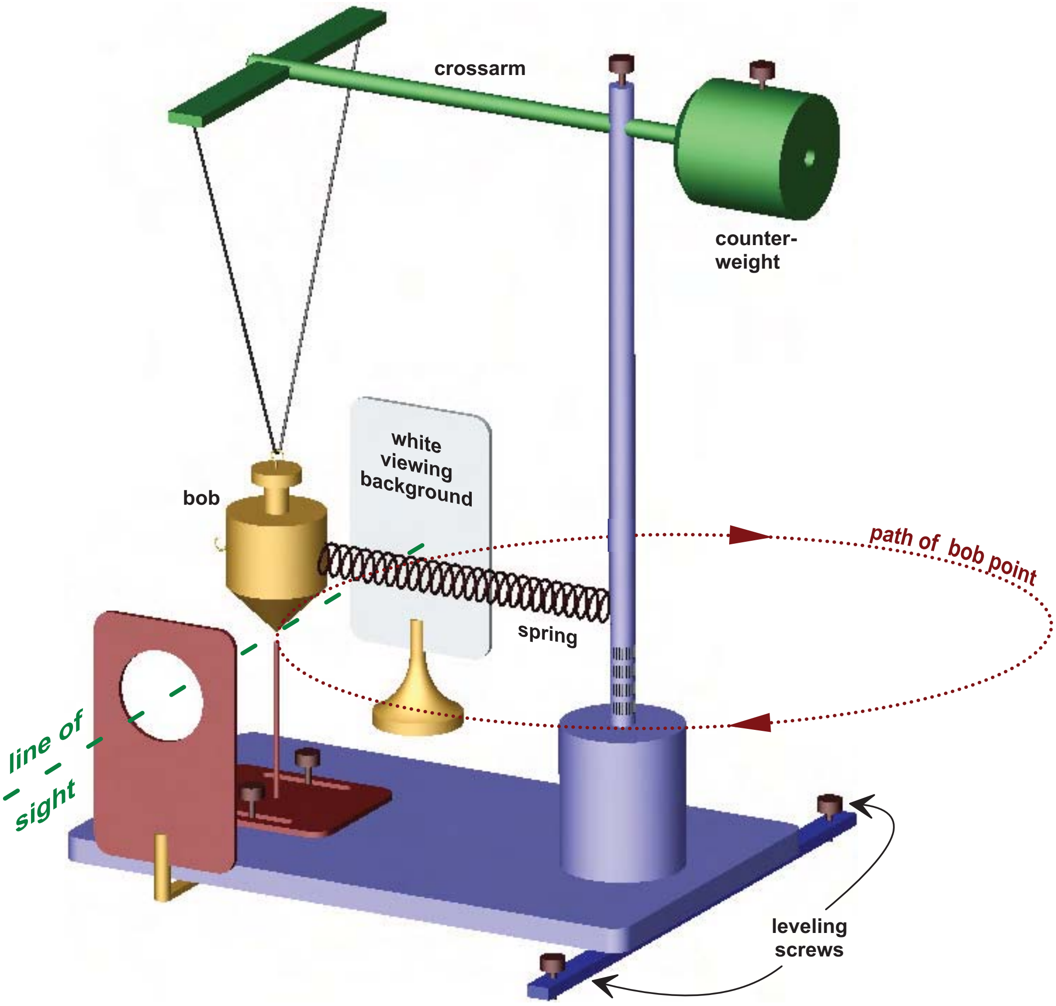


Figure 1: Centripetal force apparatus

A mass m moving with constant speed v in a circular path of radius r must have acting on it a centripetal force F, where the relationship between these quantities is

$$F = \frac{mv^2}{r}$$

Since v for this particle is given by $2\pi r/T$ or $2\pi rf$, where T is the period (seconds per revolution or s) and f is the number of revolutions per second or s^{-1}, then

$$F = \frac{m\,(2\pi rf)^2}{r} = 4\pi^2 mf^2 r \tag{1}$$

In this experiment the mass that we examine as it moves in a horizontal circle is the bob in the apparatus shown in Fig. 1. As indicated in Fig. 1, the shaft, cross arm, counterweight, bob and spring are all rotated as a unit. The shaft is rotated manually by twirling it repeatedly between your fingers at its lower end, where it is knurled. With a little practice it is possible to maintain the distance r essentially constant, as evidenced for each revolution by the point of the bob passing directly over the indicator rod, The centripetal force is provided by the spring.

The indicator rod is positioned in the following manner: with the bob at rest with the spring removed, and with the crossarm in the appropriate direction, the indicator rod is positioned and clamped by means of thumbscrews such that the point of the bob is directly above it, leaving a gap of between 2 and 3 mm.

The force exerted by the (stretched) spring on the bob when the bob is in its proper orbit is determined by a static test, as indicated in Fig. 2.

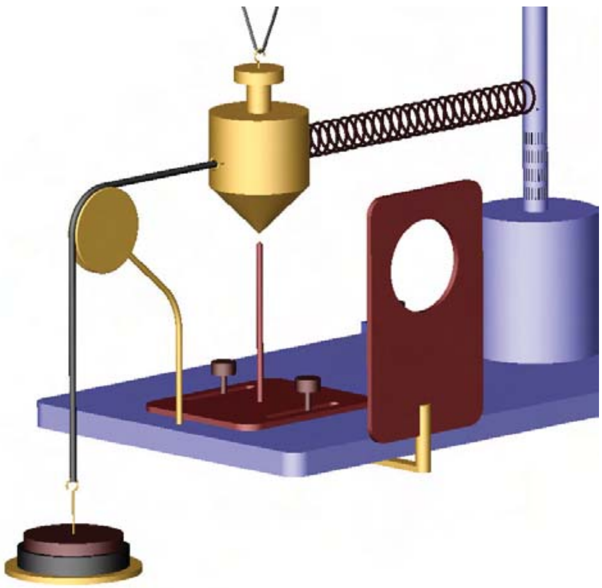


Figure 2: Static test

The mass m in Eq. 1 is the mass of the bob. A 100 g mass (slotted) may be clamped atop the bob to increase its mass.

The entire apparatus should be leveled so that the shaft is vertical. Notice that viewing should be done through the hole in the guard, against a white background.

PROCEDURE

Make any necessary adjustments of the three leveling screws so that the shaft is vertical.

The detailed procedure for checking Eq. 1 experimentally will be left to the student. At least two values of r should be used, with two values of m for each r. A method for measuring r should be thought out, the diameter of the shaft is 1.27 cm. The value of f should be determined by timing 50 revolutions of the bob and then repeating the timing for an additional 50 revolutions. If the times for 50 revolutions disagree by more than one-half second either a blunder in counting revolutions has been made, or the point of the bob has not been maintained consistently in its proper circular path. In either case, the measurement should be repeated until a consistent set of values is obtained. It is suggested that you read the next section, on results and questions, before doing the experiment.

RESULTS AND QUESTIONS

1. Exactly from where to where is r measured? Describe how you measured r.
2. Tabulate your experimental results.
3. Tabulate your calculated results for f, F from static tests, and F from Eq. 1 and the relative difference between the F's (in %), using the static F as standard.
4. Describe how to test whether the shaft is vertical without the use of a level. Why should it be exactly vertical?
5. What are the functions of the guard, the white background, and the counterweight on the crossarm?
6. Discuss your results.

Linear Momentum

APPARATUS

1. The equipment shown in Figure 1.
2. Steel sphere.
3. Waxed paper.
4. A 30cm ruler.
5. Equal arm balance and known masses.
6. Meter stick.
7. Dowel rod to free stuck sphere from block.

INTRODUCTION

The principle of conservation of linear momentum is to be tested as follows: a steel sphere is allowed to slide down the track, and immediately after leaving the end of the track plunges into a hole in a wooden block and becomes stuck within the block. The block, which is suspended by four strings, is initially at rest, but swings as a pendulum because of the impact. The momentum of the sphere before the collision is compared to the momentum of the sphere and block just after the collision.

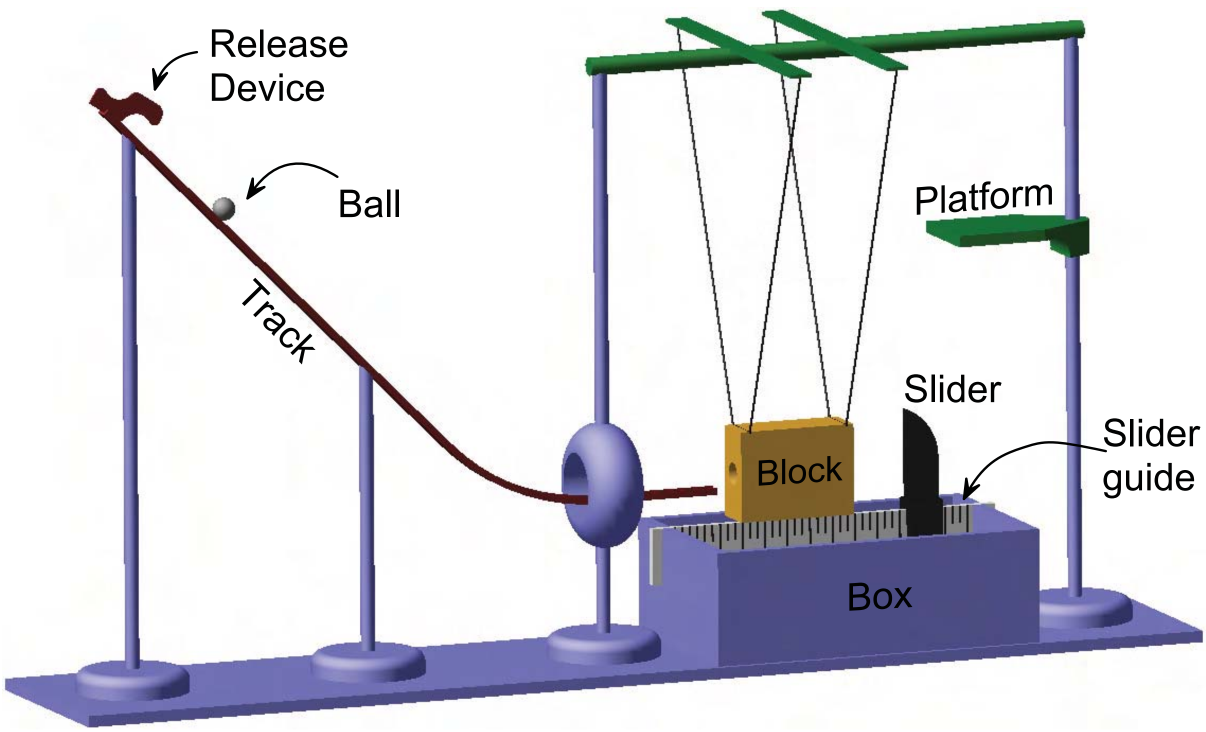


Figure 1: Entire Apparatus.

NOMENCLATURE:

m	mass of the steel sphere, [g].
M	mass of the wooden block, [g].
v	velocity of the center of the sphere as it leaves the track, [cm/s]
V	common velocity of sphere and block immediately after impact, [cm/s]
s, h, x, y, b, r	various distances, indicated in Figures 2,3, and 4, all [cm]

PROCEDURE

Part I: Determination of the Velocity of the Sphere Before Impact

Place the block on the platform, where it will he safely out of the way. Remove the slider guide and slider from the box (See Fig. 4) and clamp a strip of waxed paper to the floor of the box. Allow the steel sphere to roll down the track from its highest point. It will fall into the box and leave an imprint. The end of the track is horizontal.

Determine the height b, through which the sphere falls; be aware that the track is a channel, and the lowest point of the sphere is below the upper edges of the channel.

Make ten or more trials, and find the average value of the range r. From these data, calculate the time of flight, and the velocity of the center of the sphere as it leaves the track.

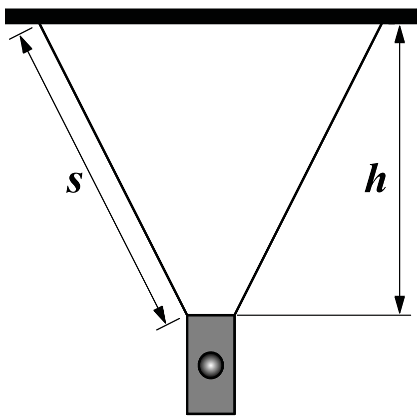


Figure 2: Front view of the block.

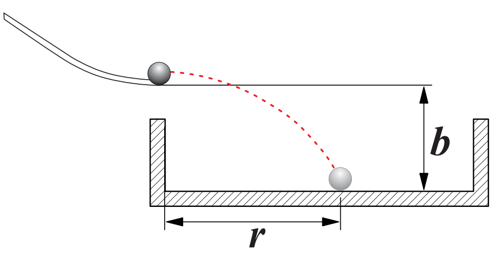


Figure 4: Determination of the steel sphere's velocity.

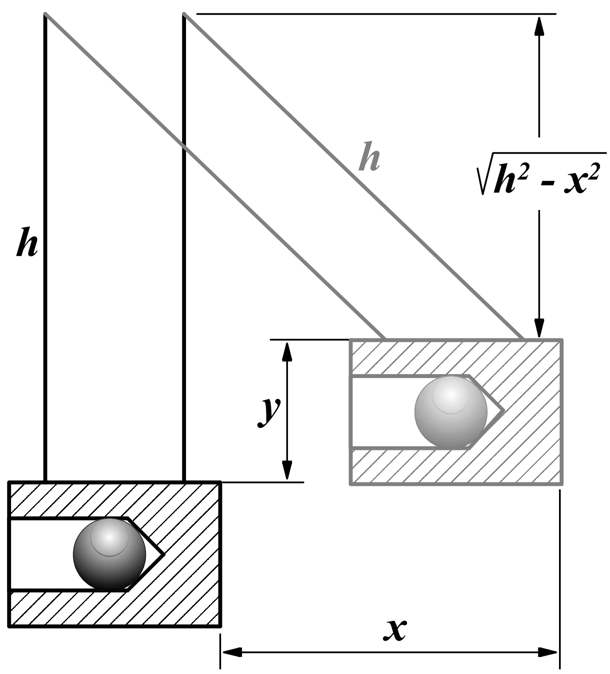


Figure 3: Side view of the block.

Part II: Determination of the Velocity of Sphere and Block After Impact.

Determine the mass of the steel sphere. Also record the mass of the wooden block, which is inscribed on the block.

Mount the slider guide and slider in the box. Suspend the block as indicated in Figure 1 at the appropriate level such that the faces of the block are parallel to the corresponding faces of the box. The block should hang freely, with a gap of about 1/8 inch between it and the track. The block must be perfectly still while awaiting the arrival of the sphere.

Measure the distance h. Notice that when the block swings, the suspension inhibits rotation (see Figs. 2 and 3). The horizontal distance x, through which the block swings after impact must be measured (see below), and the vertical distance y is calculated from (see Fig. 3):

$$y = h - \sqrt{h^2 - x^2}$$

and, from energy considerations

$$V = \sqrt{2gy} \tag{1}$$

To determine x, first position the slider so that it is barely touching the stationary block, and record this distance setting as read from the slider guide. Next, perform a few trial runs until you succeed in positioning the slider such that for ten successive impacts the block at the end of its swing sometimes just barely flicks the slider, and at other times just barely fails to reach it. Note the position of the slider and calculate x.

Calculate the linear momentum of the system before impact (mv) and after impact ($[m + M]V$), and compute the relative difference (in %). Within the limits of your experimental accuracy, is momentum conserved during the collision?

Questions (to be answered in your report):

1. Derive Equation (1), starting from general physics principles.
2. From your results, compute the fractional loss of kinetic energy of translation during impact. Disregard rotational energy of the sphere.
3. Derive an expression for the fractional loss of kinetic energy of translation in terms only of m and M, and compare with the value calculated in the preceding question. Consider the collision as a totally inelastic one.

Elasticity and Simple Harmonic Motion

APPARATUS

1. Balance and set of known masses
2. Two cylindrical springs
3. A set of five masses: 100, 200, 200, 500 and 1000 g (Hook type)
4. Upright meter stick with a movable index attached
5. Stop clock

PROCEDURE

Part I: Elasticity of a body

Since no real body is perfectly rigid the application of a force will distort it. A perfectly elastic body will return to its original form after the removal of the distorting force.

The elasticity of the cylindrical spring may be tested by comparing the position of some point on the spring before each of several loads is added with the position of the point after each load is removed.

Using the vertical meter stick observe the position of some point at the lower end of the spring. Add a load of 400 g and again note the position of the point on the spring. Increase the load by 200 g and repeat the observation. Remove the loads, one at a time, compare the position of the reference point after each load is removed with its corresponding original position.

Part II: Dependence of the distortion on the distorting force

For many bodies the force, within limits, produces a distortion which is proportional to the force. The limits within which this proportionality exists depends upon the material of which the body is made and upon the form of the body.

The body to be studied is a closely wound cylindrical spring. A force large enough to produce a permanent elongation is easily conceivable. Adjacent turns of this spring may press so tightly together that some pull is required to relieve this compressional tendency before the spring can begin to be stretched. There is possible, then, an upper and a lower limit to the force which can be applied to the spring, between which the distortion is proportional to the force.

1. Observe the position of some reference point on the lower end of the spring as in Part I. This is the zero reading.

2. Apply a load of 100 g and note the new position of the reference point.

3. Repeat, adding 100 g each time, until the total load supported is 1000 g.

4. Determine the elongation produced by each load by subtracting the zero reading from each subsequent reading.

5. Show the dependence of the elongation upon the applied force by plotting the elongation as y-axis and the corresponding total force as x-axis.

6. Determine from the curve the range of forces used in which the elongation is proportional to the force.

7. Remove the load 100 g at a time, taking the scale reading in each case. Are these readings the same, for each load, as those found above?

Part III: Force constant of the spring

The force constant of the spring is the force ΔF required to produce an elongation Δl in the spring. In symbols, this may be expressed as $k = \Delta F/\Delta l$ in units of N/m. This is a constant only for the range of forces within which the proportionality of Part II exists.

Determine an average value of the force constant of the spring from the curve plotted in Part II.

Part IV: Dependence of the period in simple harmonic motion on the vibrating mass

Consider a body, for which the distortion is proportional to the force producing it, held away from its normal position. There is now a restoring force in the body, which is proportional to the distortion. If the force applied to the body is removed, this restoring force returns the body to its normal position. However, its inertia carries it through that point producing a distortion or displacement on the other side. Now the action of the restoring force first brings the body to rest in a distorted position. The action is repeated and this simple harmonic motion continues until it is stopped by friction.

As an example, consider as the body the spring with a load of 500 g suspended from it. Reference to the data of Part II will show that now, the spring is in a condition where any additional force will produce a proportional displacement. If the load is pulled down some distance x and released, a restoring force $-k\,x$ acts on the body. As the body moves back to its equilibrium position, this restoring force diminishes. The minus sign indicates that the restoring force is opposite to the distortion. Since $-kx$ is an unbalanced force, it produces an acceleration a. From Newton's Law, $F = M\,a$, we get $-k\,x = M\,a$, where M is the mass of the system and the negative sign shows that x and a are oppositely directed. This yields

$$-\frac{x}{a} = \frac{M}{k} \tag{1}$$

The period T in simple harmonic motion is given by

$$T = 2\pi\sqrt{\frac{M}{k}} \tag{2}$$

Here, M is the mass of the vibrating system consisting of the mass suspended from the spring (500 g in the example) plus a part of the mass of the spring. It can be shown that one third of the total mass of the spring is the part effective in determining the total M.

1. Determine the period of the simple harmonic motion occurring when the load on the spring is 500 g. Determine the average time required for at least fifty vibrations.
2. Repeat with loads of 600, 700 and 800 g.
3. Measure the mass of the spring.
4. Using equation (2), calculate the period to be expected in each case. Compare them with the experimental values of the periods. Obtain both, the difference and the percent difference.
5. What percentage error would be introduced in the calculated values of T for the 500 g load and the 800 g load, respectively, if the mass of the spring were neglected?
6. Plot two curves: $T(M)$ and $T^2(M)$ (the period and the square of the period of vibration vs. the mass supported by the spring)

Part V: Dependence of the period on the amplitude

The maximum value of the displacement in simple harmonic motion is the amplitude.

Using a load of not over 600 g, try varying the initial amplitude of the vibration and note the effect on the period. Does the period depend upon the amplitude?

Buoyancy and Boyle's Law

Part A: Buoyancy and Archimedes' Principle

APPARATUS

1. Electronic balance with stand
2. Beaker
3. Metal object
4. Wooden block
5. Thread
6. Blue liquid

INTRODUCTION

The hydrostatic pressure P at a distance h below the surface of a fluid is given by

$$P = P_0 + \rho g h$$

where P_0 is the pressure at the surface of the fluid and ρ is the density of the fluid.

The hydrostatic pressure exerts a normal force on all surfaces in contact with the fluid. As a result there is a net upward force, called the Buoyant Force F_B, whose magnitude is equal to the weight of the fluid displaced, a relationship known as Archimedes' Principle.

$$F_B = \rho g V$$

where V is the volume of the object below the surface of the fluid. In this experiment, you will weigh objects in air and then measure the effect of submerging them in a fluid. A clearly labeled Free Body Diagram should be used to determine the forces on the submerged objects in order to relate your measurements to the density of the objects. If the fluid is water, assume the standard value for ρ of 1000 kg/m^3.

The electronic balance is turned on by pressing the button at the right. Pressing the button on the left quickly will change the units displayed. We will work with the gram scale. Note that this is the mass-equivalent of the force being measured, you do not have to actually multiply by the numerical value of g, leave it as a symbol and it will eventually cancel out.

PROCEDURE

Determine the density of a solid more dense than water. Weigh the metal object and then suspend it from the hook on the underside of the balance so that it is submerged in the beaker of water. This second weight, called the "apparent weight" differs from the first due to the buoyant force. Draw the corresponding Free Body Diagram and use it to determine the forces involved, and to solve for the density of the submerged object. Calculate the buoyant force and the density from your measurements. Use the table of densities on page 41 to estimate the composition of the metal. Does it appear reasonable from the appearance of the material? Explain.

An **alternate procedure** is to place the beaker on top of the scale and measure the change when the object is just submerged while being supported by the thread. Dry the metal object and weigh the beaker before (W_0) and after (W_1) the object is submerged. Do not let the submerged object touch the bottom of the beaker.

What force in the Free Body Diagram does $(W_1 - W_0)$ represent? How was this force communicated to the bottom of the beaker? (Hint: what happened to the level of water in the beaker when the object was submerged?)

Determine the density of an object less dense than water. Weigh the wood block in air. Attach the metal object to it so as to act as a "sinker". Use either method to determine the density of the wood block. Explain the procedure that you chose, including appropriate Free Body Diagrams.

Use the table of densities on page 41 to make a guess as to the type of wood provided.

Determine the density of a liquid other than water. You now have objects whose densities are known. One of them can be used as a test object to determine the density of the unknown liquid. Be sure that the object you use is as dry as possible. Use one of the earlier procedures to determine the buoyant force on the object, and calculate the density of the liquid.

Use the table of densities to make a guess as to the composition of the unknown liquid.

Question (to be answered in your report):

How large a mass would have to be placed on top of the wooden block when floating in the water so that the block would be completely submerged, i.e. its top would be level with the surface of the water?

Part B: Boyle's Law

APPARATUS

1. Sealed hypodermic syringe
2. Set of hooked weights
3. Loop of string

INTRODUCTION

Boyle's law states that for a fixed mass of gas at a constant temperature, the product of the absolute pressure p and volume V is a constant:

$$p\,V = k \tag{1}$$

With a simple apparatus we can hang masses on the syringe plunger, thereby increasing the pressure on the gas inside the syringe to a value above atmospheric pressure. The volume can be read from the scale on the side of the syringe. The scale is in units of cc which means cm^3. If the pressure added by the weight of the masses is p_{add} and the atmospheric pressure is p_{atm}, then the resulting total pressure is $p_{total} = p_{atm} + p_{add}$.

PROCEDURE

1. Record the current atmospheric pressure and temperature as indicated by your instructor. (If the current temperature is not available, assume 68°F = 20°C.) Atmospheric pressure may be given in mbar or bar, where 1 bar = 10^5 Pa. Convert to Pascals.

2a. Check that the syringe is held firmly in the clamp and is vertical. Examine the scale and record the volume V_0, indicated by the bottom edge of the plunger.

2b. Record the sensitivity of the scale, that is, the smallest quantity that can be read or estimated on it. We will use this value as the uncertainty of our volume measurements.

2c. How many digits of V_0 are actually significant? (See the discussion on significant figures in the introduction of the lab manual.)

3. We use a string to hang masses on the plunger. The weight of this mass acting on the area of the plunger will increase the pressure of the trapped air and therefore change the volume. We can simplify the calculation of added pressure for each value of mass by considering the pressure p_1, at a load of 1 kg . There are four calculations:

(i) the weight of 1 kg, W_1,
(ii) the area of the face of the plunger A, (radius of the plunger = 0.715 cm),
(iii) the pressure added by the weight, $p_1 = W_1/A$, and
(iv) conversion of the units to the standard unit of pressure, the Pascal. (1 Pa = 1 N/m^2)

Check : p_1 should be about 3/4 of an atmosphere (1 atmosphere = 1.013×10^5 Pa).
Finally, trim p_1 to the number of significant figures found in 2c. (One extra digit may be kept as a guard digit to avoid round-off problems.)

4. Use the headings below to prepare a table for your data. Note the second line of the heading that contains the multiples and units to be used.

Mass M [kg]	Volume V [10^{-6} m^3]	p_{add} [10^5 Pa]	p_{total} [10^5 Pa]	$1/p_{total}$ [10^{-5} Pa^{-1}]
0	V_0	0	p_{atm}	
...	...			

Your first entry will be for Mass = 0, and Volume = V_0 as found in 2a. Note that the units have to be converted to m^3 (1 cm = 10^{-2} m $\longrightarrow$ 1 cm^3 = 10^{-6} m^3). p_{add} will be zero, and therefore p_{total} is just equal to the atmospheric pressure.

5. Use the string to hang weights on the plunger. Use values of 500, 700, 1000, 1200 and 1500 g. Wait at least a minute after each weight is added, so that the gas can come back to room temperature. Increase the waiting time at the larger loads, so that the gas can return to room temperature after being compressed. Use the time to complete some of the calculations outlined below. Record the volume and complete the line in the table. Remember that $p_{add} = [M/(1\ \text{kg})] \times p_1$, where p_1 is the additional pressure due to a 1-kilogram weight. Trim p_{total} of any digits that are not significant.

ANALYSIS

1. We will use Boyle's law in the form:

$$V = k\,(1/p) \tag{2}$$

Calculate all the reciprocals and put them into the last column of the table.

2a. Plot the graph of V vs. $1/p_{total}$. The origin should be included, although it is not a data point. Be sure that your scales allow the graph to occupy most of the page. (We are following the usual practice of plotting the dependant variable on the y-axis.)

2b. Draw the single straight line that best represents the data. Use a transparent straight edge (like a plastic ruler) to help fit the line. The line should be drawn completely across the graph.

2c. Choose two points on the line (not data points) that are widely separated to use in calculating the slope. Note that the slope will have units. Record this value as k_{slope}.

3. The graph of equation 2 would go through the origin. With experimental data, the straight line usually comes close to, but misses the origin. Determine the positive intercept with either axis. Assume that the uncertainties in the value of these intercepts are 0.2 cm^3 and $0.04 \times 10^{-5}\text{Pa}^{-1}$, respectively.
We can see whether this intercept is consistent with 0 by calculating the uncertainty ratio, your intercept value divided by the appropriate uncertainty value. A small ratio (less than 2) indicates good agreement. Large values (greater than 5) means disagreement. Intermediate values can be described as 'fair' or 'poor' agreement and usually require further study.

4. The constant k can also be determined from the Ideal Gas Law, $p\,V = n\,R\,T$, where n is the number of moles of gas ($\rho V_1/MW$), R is the gas constant (8.31 J/mol·K) and T is the absolute temperature in Kelvin. V_1 is the volume measured at a load of 1 kg and ρ is the corresponding density. For air, 80% $^{14}N_2$ and 20% $^{16}O_2$, the molecular weight MW $= 29 \times 10^{-3}$ kg/mol, and $\rho = 2.13$ kg/m^3 at the 1-kg load and room temperature. Calculate $n\,R\,T$ from the data given, and compare with k_{slope}, assuming an uncertainty in k_{slope} of 0.2×10^5 Pa·cm^3.

Questions (to be answered in your report):

1. (a) What curve would equation 1 describe in a graph of p vs. V?
   (b) How could we graph our data so as to obtain a straight line with slope k ?

2. How does your value of k_{slope} agree with $n\,RT$? The uncertainty ratio here is

$$\frac{|k_{slope} - n\,RT|}{\text{uncertainty in } k_{slope}}$$

3. If the uncertainty in the slope is ΔV (from procedure 2b.) divided by the range of your $(1/p)$ values, what is your uncertainty in k_{slope} ?

Density Table

Metal	ρ **[g/cm³]**
Aluminum	2.7
Brass (ordinary yellow)	8.40
Bronze - phosphor	8.80
Copper	8.90
Gold	19.3
Iron - wrought	7.85
Iron - gray cast	7.1
Lead	11.3
Steel	7.8
Tungsten	19.3
Zinc - wrought	7.2
Balsa wood (oven dry)	0.11 ... 0.14
Ebony	1.11 ... 1.33
Oak	0.6 ... 0.9
Pine - white (oven dry)	0.35 ... 0.50

Liquid	ρ **[g/cm³]**
Alcohol, Methyl	0.80
Carbon Tetrachloride	1.60
Gasoline	0.68
Mercury (20°C)	13.55
Water (0°C)	0.999
Water (4°C)	1.000
Water (15°C)	0.997
Water (100°C)	0.958

Stone	ρ **[g/cm³]**
Granite	2.7
Limestone	2.7
Marble	2.6 ... 2.8
Mica schist	2.6
Sandstone	2.1 ... 2.3